1. Introduction: Earth’s tropical atmosphere and ocean

Recognition of the nature including our planet Earth was started at first as description of locality of ground and sky by geography and astronomy (geodesy), and then understood theoretically using generalized laws of physics. Greek and Roman scientists pioneered the first category, and among them Eratosthenes (c. 275 – c.194 BC), Hipparchus (c.190 – c.120 BC) and Ptolemaeus (c. AD 83 – c. 168) recognized surely the Tropic of Cancer and the Tropic of Capricorn as the northern and southern latitudinal limits over Earth where Sun can arrive in the zenith on each solstice, or those of Sun over the celestial sphere (cf. Chapter 4, Section 6.1). The low-latitude region around the equator between the both Tropics became called “tropics” geographically until the great voyage ages.

So far not a few textbooks concerned theoretical, physical or dynamical meteorology, but most of them described almost only on the mid-latitudes; such as geostrophic winds, extratropical cyclones, fronts and their practical application tools called weather maps. However, many of these concepts are almost useless in the tropics. Pages of chapters of tropical meteorology are used mainly for tropical storms, although they appear actually in subtropics and very rarely in the central tropics near the equator (cf. Section 6.3). In the equatorial region or low latitudes with solar-energy input (excess) and almost horizontal Earth’s rotation, forced convective motions in vertical planes are more essential rather than unstable horizontally vortical motions dominant in the middle/latitudes (Chapter 4).

The two aspects mentioned above are both still continued at present. On one hand, because of delayed/limited establishment of observation network over developing countries and broader oceans, improvement of the geographical description aspect is still necessary. On the other hand, as well as the above-mentioned dynamical features, interannual/intraseasonal interactions with open oceans and water cycles with rainforest lands have been requesting establishment of the physical aspect of tropical meteorology as a rather new paradigm. This is the author’s major motivation to write this book.

Borderless combination of the two aspects is also required by recent computer-network innovations, sustainability crisis (environmental damages with continuous development) and extraterrestrial/extrasolar knowledge expansions. Now geographical observers study numerical physical model output, and atmospheric physicists study advanced geographical observations and geography of other planets.

Only a few physical principles are commonly used for any subsystems (atmosphere, hydrosphere and oceans), and interdisciplinary approaches necessary for climate studies are optimistic. Quantitative descriptions and predictions using mathematics and computers as tools are more efficient assessment/operations for environment, disaster. Conservation equations of mass, momentum and energy (entropy) are shown in Chapter 2. Both the two major forcing causes, gravitation and radiation, obey inverse-square laws of distance l:

$$\propto \frac{1}{l^2} \tag{1.1}$$

as discovered by Sir Isaac Newton (Fig. 1.1)[3], although their appearances on Earth are somewhat different: the lunar/solar gravitational forcing generates oceanic and planetary tides with two peaks per rotation, and solar radiative

[3]The inverse square law of distance is derived from a potential given by the solution of Poisson equation (Chapter 4).

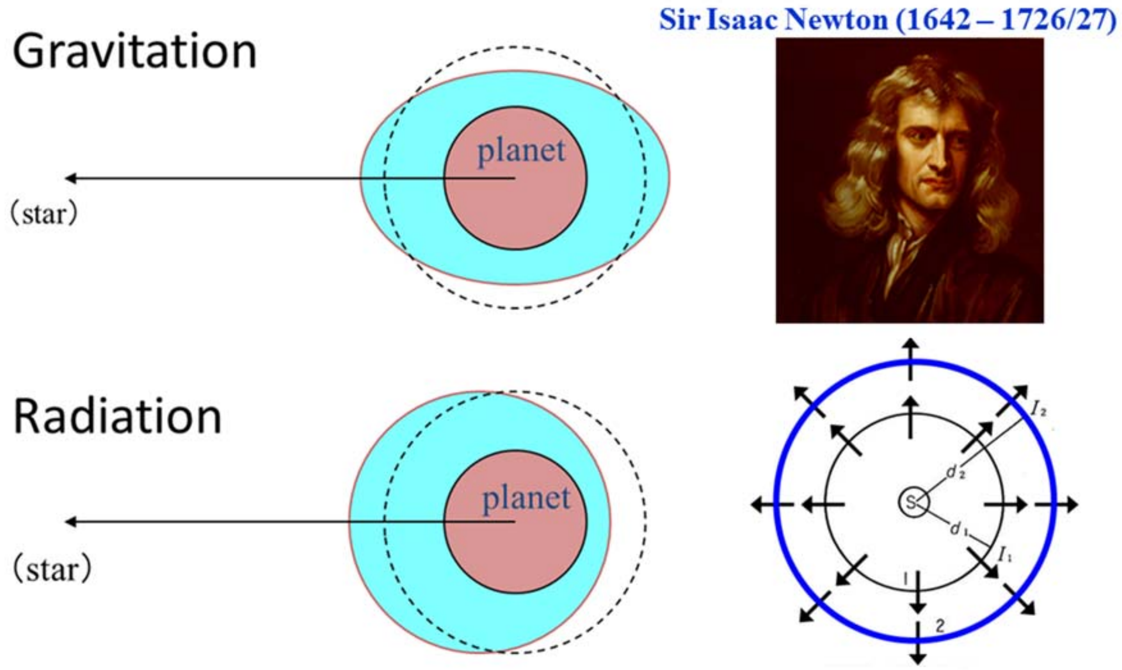

Fig. 1.1 Two major forcing causes; gravitation and radiation. Left: Schematic comparison of the two types of tidal forcing (left; the latter is a net between stellar and planetary radiations). Right top: Portrait of Newton (http://www.newton.ac.uk/about/art-artefacts/newton-portrait). Right bottom: The inverse square law of distance for radiation.

forcing produces day/night or atmospheric tide with only one peak[4].

The gravitation is acted between two bodies, and its intensity is proportional to their masses M and m:

$$F = -G\frac{Mm}{l^2}, \tag{1.2}$$

where $G = 6.6741\times10^{-11}$ Nm^2/ kg^2 (=m^3 kg^{-1}s^{-2}) is the gravitational constant, and the negative sign indicates it acts on the direction of negative l. If we insert it in Newton's equation of motion put $F = -mg$, then

$$g = \frac{GM}{l^2} \tag{1.3}$$

is called gravity acceleration. For example:

For Earth ($l = l_{\mathrm{ES}} \equiv 1.496\times10^{11}$ m) around Sun ($M = 1.988\times10^{30}$ kg): $g_{\mathrm{sun}} = 5.928\times10^{-3}$ m/s^2;

For Earth ($l = l_{\mathrm{EM}} \equiv 3.844\times10^{8}$ m) from Moon ($M = 7.346\times10^{22}$ kg): $g_{\mathrm{moon}} = 3.318\times10^{-5}$ m/s^2;

For a body ($l = a \equiv 6.371\times10^{6}$ m) on Earth ($M = 5.972\times10^{24}$ kg): $g = 9.820$ m/s^2,

where l is taken as the mean orbital or planetary radius. The former two are negligibly smaller than the last one, but each variation between the maximum (at subsolar or sublunar point) and the minimum (each antipode) becomes

$$\Delta g_{\mathrm{sun}} = g_{\mathrm{sun}}\left(1-\frac{a}{l_{\mathrm{ES}}}\right)^{-2} - g_{\mathrm{sun}}\left(1+\frac{a}{l_{\mathrm{ES}}}\right)^{-2} \approx \frac{4a}{l_{\mathrm{ES}}} g_{\mathrm{sun}} = 1.010\times10^{-6}\ \mathrm{m/s^2};$$

$$\Delta g_{\mathrm{moon}} \approx \frac{4a}{l_{\mathrm{EM}}} g_{\mathrm{moon}} = 2.199\times10^{-6}\ \mathrm{m/s^2}; \quad \Delta g = 0,$$

[4] For global atmospheric tides the semidiurnal component is dominant because of vertical propagation condition (Chapter 5), whereas local land-sea contrast generates dominant diurnal cycle (Section 6.1).

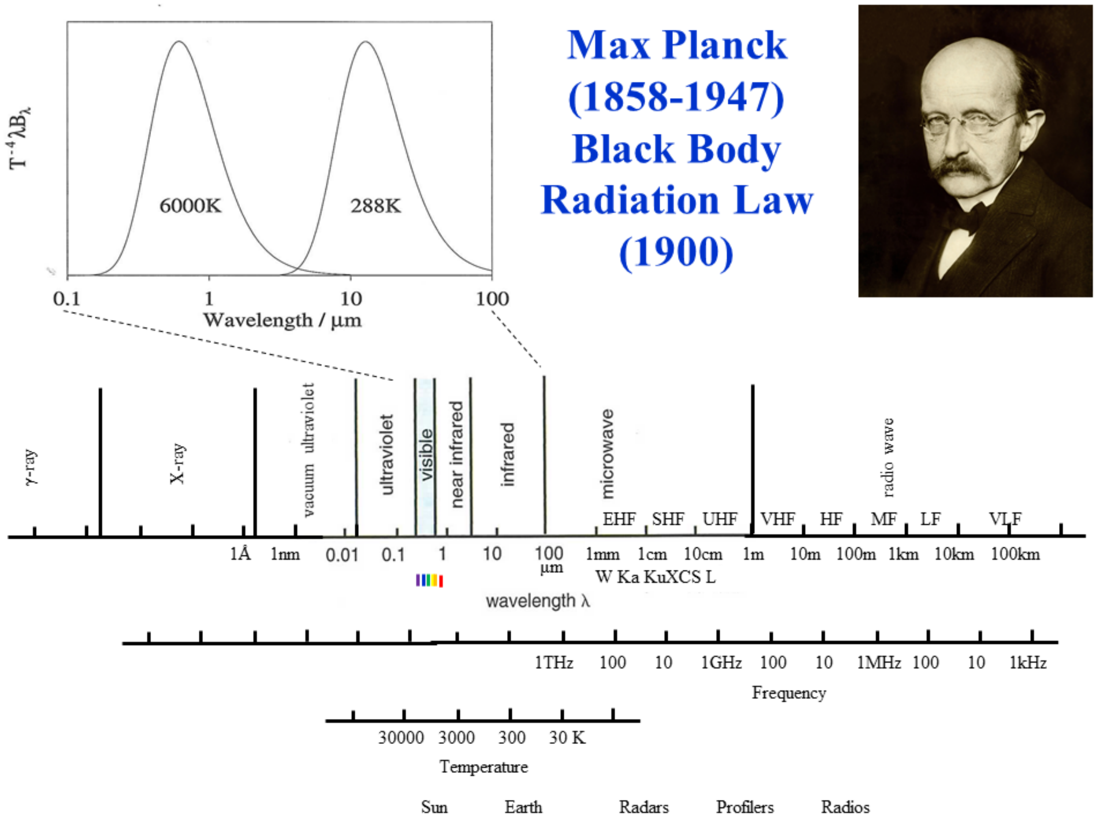

Fig. 1.2 The black body radiation law (top left; Andrews 2000) by Planck (top right; http://www.gahetna.nl/collectie/afbeeldingen/fotocollectie/zoeken/weergave/detail/start/2/tstart/0/q/zoekterm/Planck), and classification of electromagnetic waves (bottom).

which explain that a gravitational (oceanic) tide is generated mainly by Moon and partly by Sun.

The radiation is emitted from the surface of a body, and its spectrum (energy per unit area and unit time of electromagnetic waves per unit wavelength) is given idealistically (as a black body) by the Planck law (Fig. 1.2):

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5\{\exp[hc/(\lambda kT)] - 1\}}, \tag{1.4}$$

where $h = 6.6261{\times}10^{-34}$ Js (= m^{2}kg/s) is the Planck constant, $k = 1.3806{\times}10^{-23}$J/K is the Boltzmann constant, $c = 2.9979{\times}10^8$ m/s is the speed of light (in vacuum), T is temperature of the body surface (as a black body), and λ [m] is wavelength (= c/ν, where ν [Hz = 1/s] is frequency). Integration of (1.4) over contribution on a flux density (energy per unit area and time) from the outside hemisphere and the whole wavelength gives the Stefan-Boltzmann law:

$$B(T) \equiv \int_{2\pi} \int_0^\infty B_\lambda(T) d\lambda \Big|_{\text{upward}} d\omega = \sigma T^4, \ \ \sigma \equiv \frac{2\pi^5 k^4}{15 h^3 c^2}, \tag{1.5}$$

where $\sigma = 5.67{\times}10^{-8}$ Wm^{-2}K^{-4} is the Stefan-Boltzmann constant, and differentiation of (1.4) with respect to λ gives Wien's displacement law:

$$\frac{\partial B}{\partial \lambda} = 0 \qquad \therefore \ \left(5 - \frac{hc}{k\lambda T}\right) e^{hc/k\lambda T} = 5 \qquad \therefore \ \lambda T = 2857[\mu\text{m K}] \tag{1.6}$$

(Exercise 1-5). By (1.6) the solar surface of about 5,800 K radiates mainly visible light with wavelengths around 0.5 µm, and the Earth's surface of about 15ºC = 288 K emits mainly infrared radiation with about 10 µm. The solar

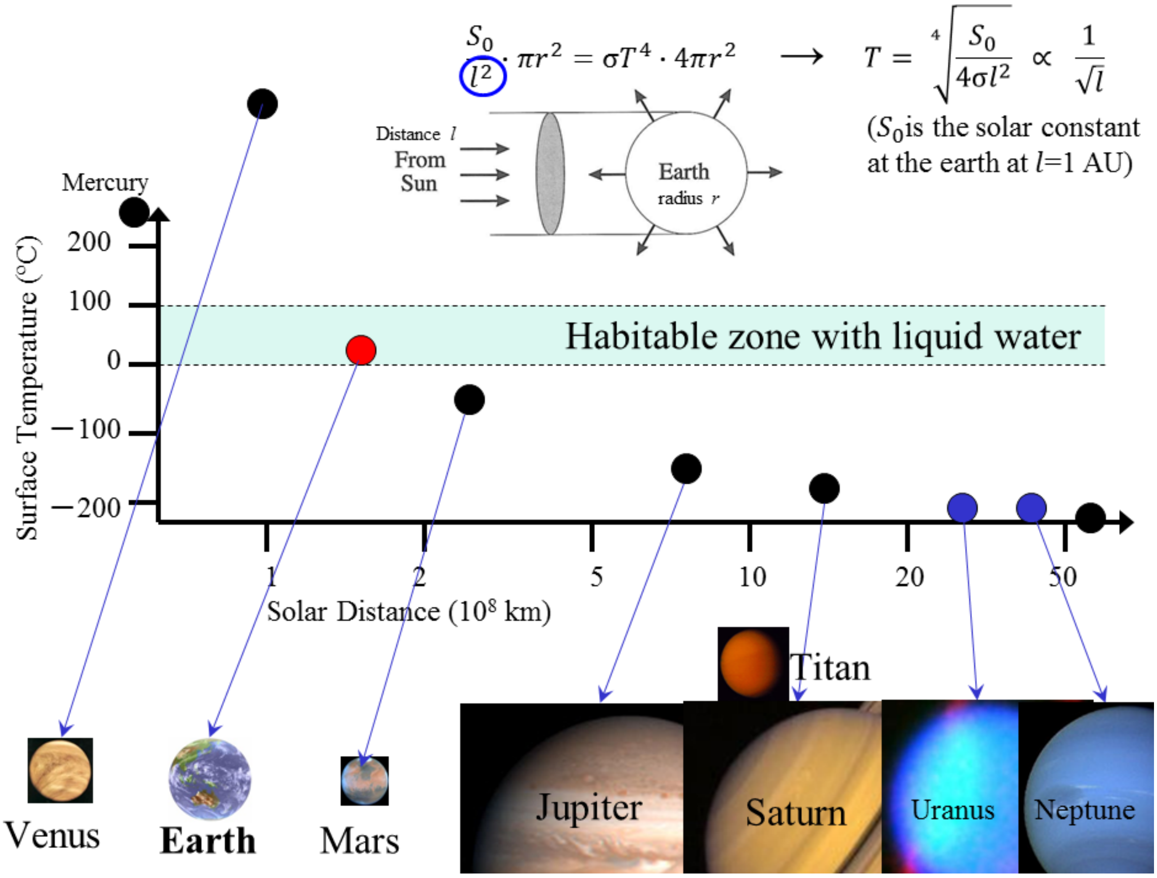

Fig. 1.3 The zero-dimensional radiative equilibrium (top formula with schematic figure) and surface temperature-solar distance relation for planets (photos: Earth by JMA-GMS and Kochi U-Weather Home (http://weather.is.kochi-u.ac.jp/index-e.html) : Uranus by NAOJ-Subaru and Kobe U-CPS (http://subarutelescope.org/Pressrelease/2002/02/21/index.html), the others by NASA (http://solarsystem.nasa.gov/planets/solarsystem).

temperature is actually estimated from the solar radiation using (1.4) (or (1.5) and (1.6)) inversely.

By the inverse-square law the solar radiation flux density is given by S_0/l^2, where $S_0 =$ 1.366×10^3 W/m^2 is the solar constant (as observed at Erath) and l is a non-dimensional solar distance in terms of the astronomical unit (AU= $l_{\mathrm{ES}} =$ 1.496×10^{11} m). Incident radiative energy (per unit time) at a planet with radius r and solar distance l is given by S_0/l^2 multiplied with the cross-sectional area πr^2. The planetary radiative energy emission (per unit time) is given by the flux density σT^4 following the Stefan-Boltzmann law (1.5), multiplied with the planetary surface area $4\pi r^2$. Therefore the simplest form of zero-dimensional radiative equilibrium is given by the balance of these incident and emitted energies:

$$\frac{S_0}{l^2} \cdot \pi r^2 = \sigma T^4 \cdot 4\pi r^2 \qquad \therefore\ T = \left[\frac{S_0(1-A)}{4\sigma l^2}\right]^{1/4}, \tag{1.7}$$

where we have included a reflection (albedo) A, or the so-called parasol effect, for the incident solar energy. This implies $T \propto 1/\sqrt{l}$, which seems to be satisfied by planets of the solar system (Fig. 1.3). However, Venus has higher temperature, which is due to the so-called greenhouse effect, which will be theoretically discussed in Section 3.1. Even for Earth (1.7) gives T = 278 K or 254 K for A = 0 (unrealistic) or 0.3 (observed), respectively (Exercise 1), and the actual Earth's temperature is warmer than estimation from (1.7) due to the greenhouse effect.

Actual Earth's energy budget is more complex, including water cycle as well as parasol and greenhouse effects

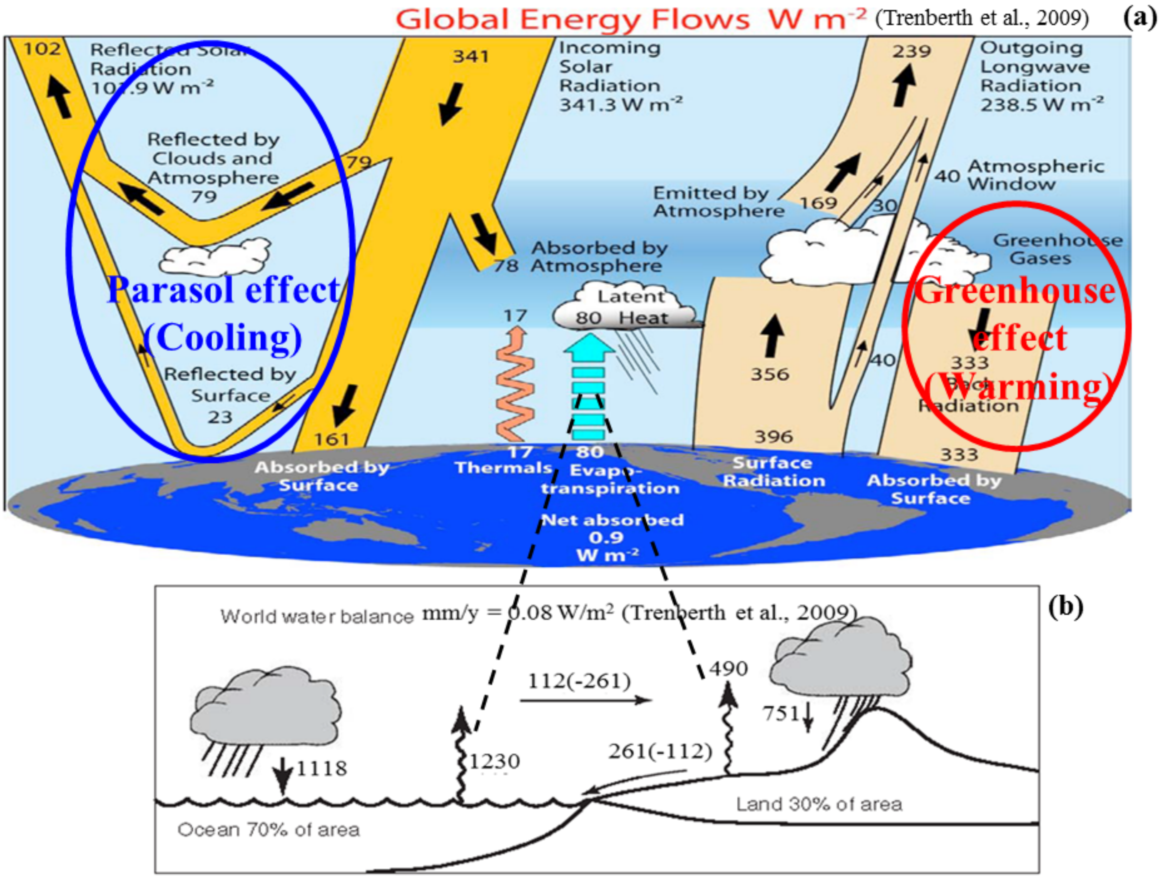

Fig. 1.4 Global (a) energy and (b) water budgets (Trenberth et al., 2009), which is also adopted in IPCC reports.

(Fig. 1.4). The water cycle penetrates ocean, atmosphere and land with phase change (Sections 3.3 and 6.2), and the process of evaporation (from land and ocean) and condensation (cloud generation in the atmosphere) is a heat transport from the Earth's surface to the atmosphere. Water vapor is a greenhouse gas, and clouds concern also the parasol effect. Namely water is related to both warming and cooling. Earth is only one planet critically within the so-called "habitable zone" with liquid water (Fig. 1.3).in the solar system, and the tropics is the main region of water cycle (tropopause cold trap, cloud generation, and rainfall). Solar radiation also encounters various scattering processes by molecules and cloud particles (crystals and droplets) in the atmosphere (Fig. 1. 5).

The tropical (or equatorial) atmosphere, hydrosphere and oceans have many peculiar characteristics which are rather different from the extratropics (or middle/higher latitudes). The Earth's rotation axis becomes horizontal, and the Coriolis force becomes smaller, although the meridional gradient (or the β-effect, see Chapters 4 and 5) is relatively larger. Stronger solar heating with weaker annual cycle and weaker synoptic-/meso-scale cyclones induce relatively stronger diurnal cycles, which is another effect of the Earth's rotation. Broad oceans with warmer waters generate active evaporation, convective clouds, latent heating and rainfalls. Unlike wind (around cyclones) making clouds in extratropics, clouds make wind in tropics.

In addition the Earth's history has made peculiar features of the tropics. The universe started 13.7 billion years ago, and the solar system including Earth and the other planets were born 4.5 billion years ago. In its early history Earth had continents, oceans and lives, but the lives landed only 400 million years ago after sufficient amount of

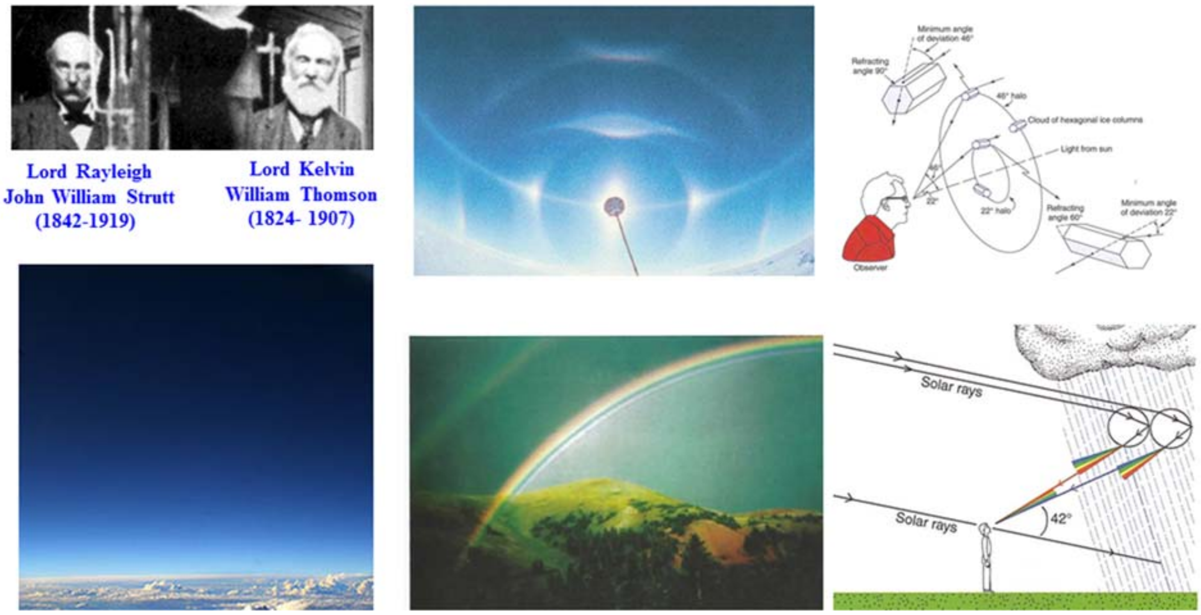

Fig. 1.5 The two great British classical physicist: Rayleigh and Kelvin (left top) and blue sky die to atmospheric molecular scattering named Rayleigh (left bottom; also see fluid dynamical contribution in Section 6.4), whereas Kelvin contributed to thermodynamics (as using his name for temperature unit) and fluid mechanics (cf. Section 5.1). Halo seen around Sun by refraction of sunlight by cloud ice crystals (middle/right top), and rainbow seen on standing against Sun by raindrops (middle/right bottom).

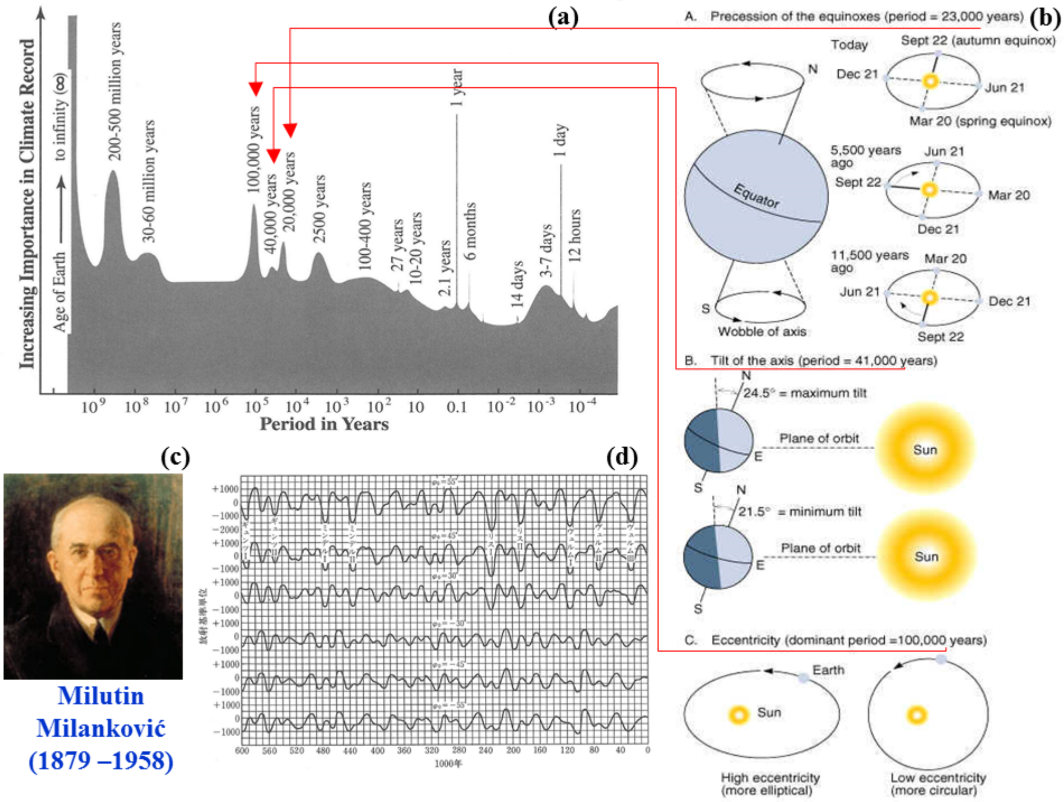

Fig. 1.6 (a) Geological-scale climate variability (Mitchell, 1976; Peixoto and Oort, 1992; Lunine, 1999), (b) Earth's astronomical variability (http://www.detectingdesign.com/milankovitch.html), and (c) Milanković's (http://b.static.trunity.net/files/ 120401_120500/ 120456/Milankovitch.jpg) (d) calculation (Milanković, 1941).

oxygen produced the ozone layer intercepting solar ultraviolet rays. We human being appeared very recently not much older than 5 million years ago. Throughout Earth's history climate changed largely, for example due to the

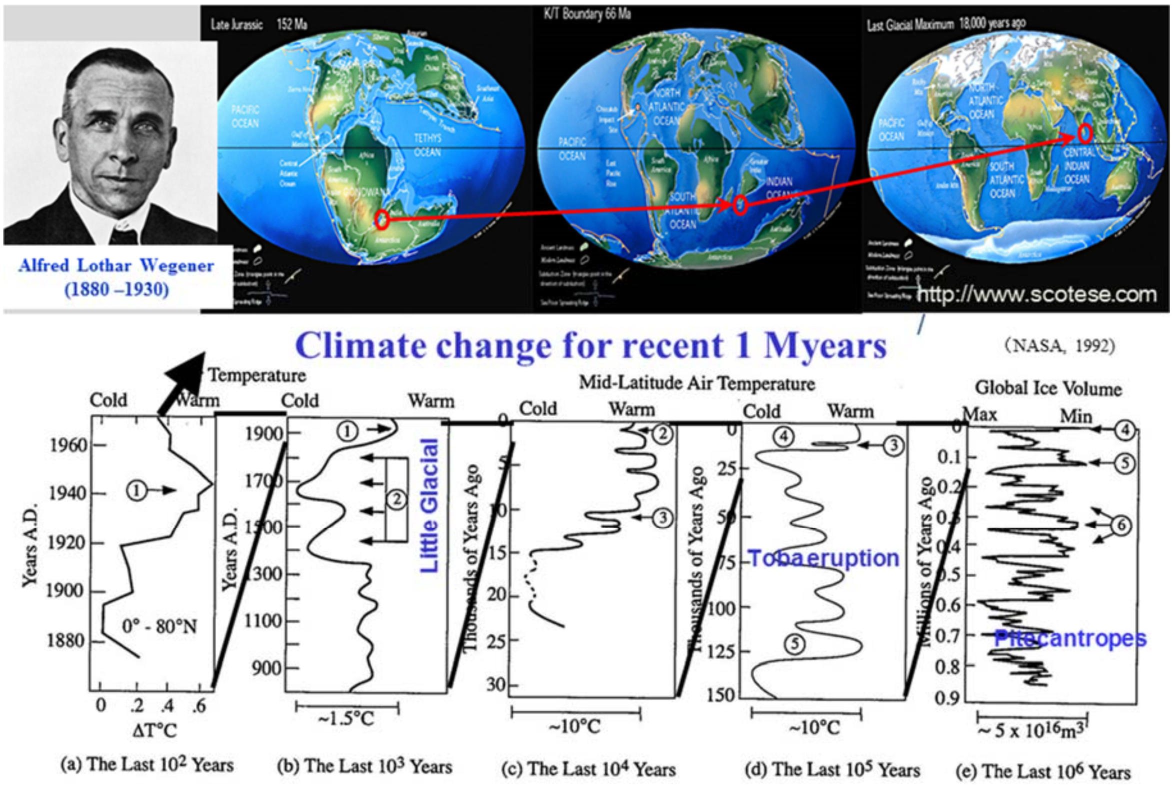

Fig. 1.7 Continents (and ocean) displacements for recent 150 Myears (top; http://www.scotese.com/) advocated first by Wegener (http://www.bildindex.de/bilder/fm426294a.jpg), and climate changes for recent 1 Myears (bottom; Hartmann, 1992).

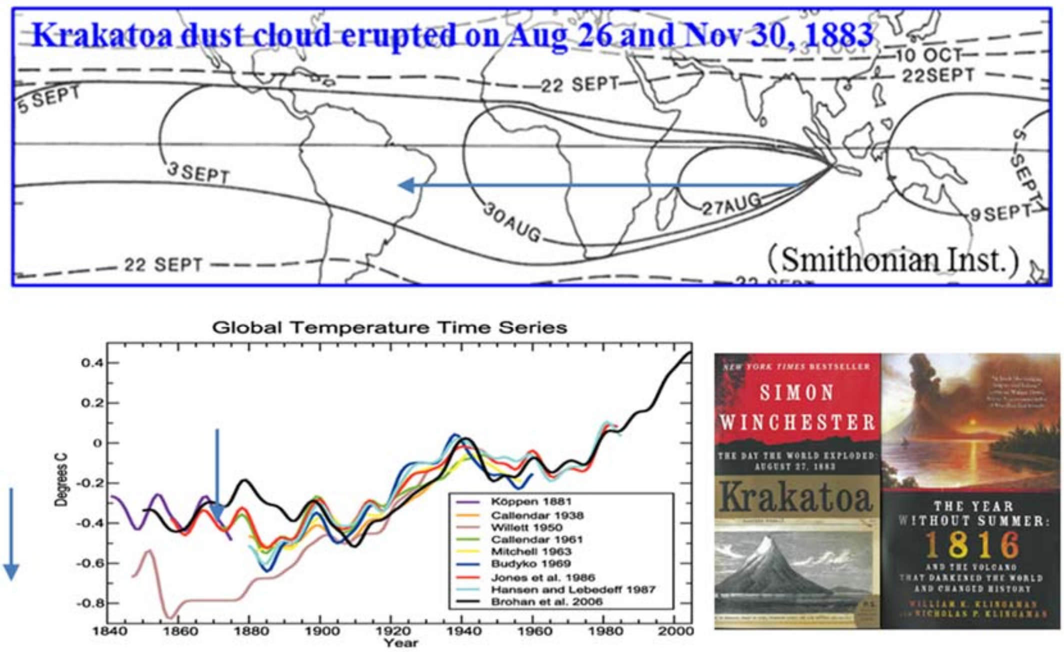

Fig. 1.8 Dispersion of Krakatoa ashes in 1883 (top; Barry and Chorley, 2003), and cooling (bottom left, https://www.ipcc.ch/) by eruptions of Tambora in 1816 (Klingaman and Klingaman, 2013) and Krakatoa (Winchester, 2003).

Earth's astronomical motion (precession and changes of obliquity, and eccentricity) (Fig. 1.6), plate-tectonic displacements of continents and oceans (Fig. 1.7) and volcanic eruptions (Fig. 1.8). The obliquity is directly related to the latitudes of tropics, and the plate tectonics and volcanoes have made peculiar geographic conditions over the tropics, such as the maritime continent. The volcanic eruptions may cause parasol-effect cooling, even when the greenhouse-effect warming proceeds such as recent centuries, and the actual situations are strongly dependent on the tropical general circulations not only in the troposphere but also in the stratosphere.

The fact that the tropics without established meteorology covers a half of Earth's surface is one of the most fatal issue in understanding and predicting the global climate of Earth. So-called "capacity building" in international cooperation is usually considered as that developed countries transfer their knowledges/technologies to developing countries, but it is impossible in tropical meteorology, because it has never established in any developed countries.

Exercise 1

(1) Knowing the solar constant (solar radiation intensity at the top of the atmosphere) as S_0=1.37×10^3 W/m^2 (where 1 W = 1 J/s) and the sun-earth distance (1 AU) as l_0=1.5×10^{11} m, calculate the following.

(a) Earth's temperature in the simplest radiative equilibrium with A = 0. Use the Stefan-Boltzmann constant: σ=5.67×10^{-8} W/m^2/K^{-4}.

(b) Actually about 30% of S_0 should be returned to the space (namely A = 0.3). Then how large the equilibrium temperature? What about the difference from actual earth's temperature?

(c) Estimate equivalent temperature on Venus at 0.728 AU and on Mars at 1.524 AU, and compare it with observational evidence in the previous slide.

(d) Estimate the surface temperature of the sun with the radius 7.0×10^8 m. (a little lower than the actual value)

(2) Integrating and differentiating (1.4), derive (1.5) and (1.6).

Answers:

(1)(a) $T = \sqrt[4]{S_0/4\sigma l^2} \fallingdotseq 278\text{K}$ (Note that l is taken by AU. F or earth, $l = 1.$)

(b) $\sqrt[4]{1-0.3} \fallingdotseq 0.915$. $278\text{K} \times 0.915 = 254\text{K} = -19$°C (too cold! Green house effect must be considered. → Chap.3)

(c) Venus: $\sqrt[4]{1/(0.728)^2} \fallingdotseq 1.172$. $278\text{K} \times 1.172 = 325\text{K}$ (lower than actual! Due to greenhouse effect)

Mars: $\sqrt[4]{1/(1.524)^2} \fallingdotseq 0.729$. $278\text{K} \times 0.729 = 202\text{K}$

(d) $7.0\times10^8\text{m}/\ 1.5\times10^{11}\text{m} = 4.7\ \times10^{-3}$ AU. $\sqrt[4]{1/(4.7\times10^{-3})^2} \fallingdotseq 10.45$

$278\text{K} \times 10.45 = 2905\text{K}$ (lower than actual; due to neglect of radiation from below the solar surface)

(2) Assuming isotropy, the upward component is obtained by multiplying $\cos\varsigma$, where ς, is the zenith angle, and the sold-angle integral with $d\omega$ the upward hemisphere is rewritten as small-circle integral with $2\pi\sin\varsigma\, d\varsigma$.

$$B(T) = \int_0^{\pi/2}\int_0^{\infty} 2\pi B_\lambda(T)\cos\varsigma\sin\varsigma\, d\lambda d\varsigma = 2\pi\int_0^{\pi/2}\cos\varsigma\sin\varsigma\, d\varsigma \cdot \int_0^{\infty} B_\lambda(T)d\lambda$$

In the first integral we put $x \equiv \sin\varsigma$, and in the second $y \equiv hc/kT\lambda$. Then $dx \equiv \cos\varsigma\, d\varsigma$, and $dy \equiv -(hc/kT\lambda^2)d\lambda$.

$$B(T) = 2\pi\int_0^1 x dx \cdot \frac{2k^4T^4}{h^3c^2}\int_0^{\infty}\frac{y^3}{e^y-1}dy \equiv \sigma T^4$$

The second integral may be obtained by a complex integral as $\int_0^{\infty} y^3/(e^y-1)\, dy = \pi^4/15$, and the constant becomes

$$\sigma = 2\pi\int_0^1 x dx \cdot \frac{2k^4}{h^3c^2}\int_0^{\infty}\frac{y^3}{e^y-1}dy = \frac{2\pi^5k^4}{15h^3c^2} = 5.67\times10^{-8}\ [\text{Wm}^{-2}\text{K}^{-4}],$$

using the Boltzmann (absolute gas) constant k=1.38×10^{-23} J/K, the Planck constant h=6.63×10^{-34} J s, and the light speed c=3.00×10^{8} m/s. Finally, the differentiation of the Planck function is solved numerically as

$$\frac{\partial B}{\partial \lambda} = 0 \quad \therefore \quad \left(5 - \frac{hc}{k\lambda T}\right) e^{hc/k\lambda T} = 5 \quad \therefore \quad \lambda T = 2857[\mu\mathrm{m\ K}]$$