2. Conservation laws and basic equations

Actual matter = Σ molecule

- Density = Σ molecular mass / unit volume
  = molecular mass × number / unit volume

- Pressure = Σ molecular momentum / unit time / unit area
  = molecular force / unit area

- Temperature = Σ molecular kinetic energy
  / molecular number / Boltzmann constant

Mass conservation and fluid continuity

Leonhard Euler (1707～1783)

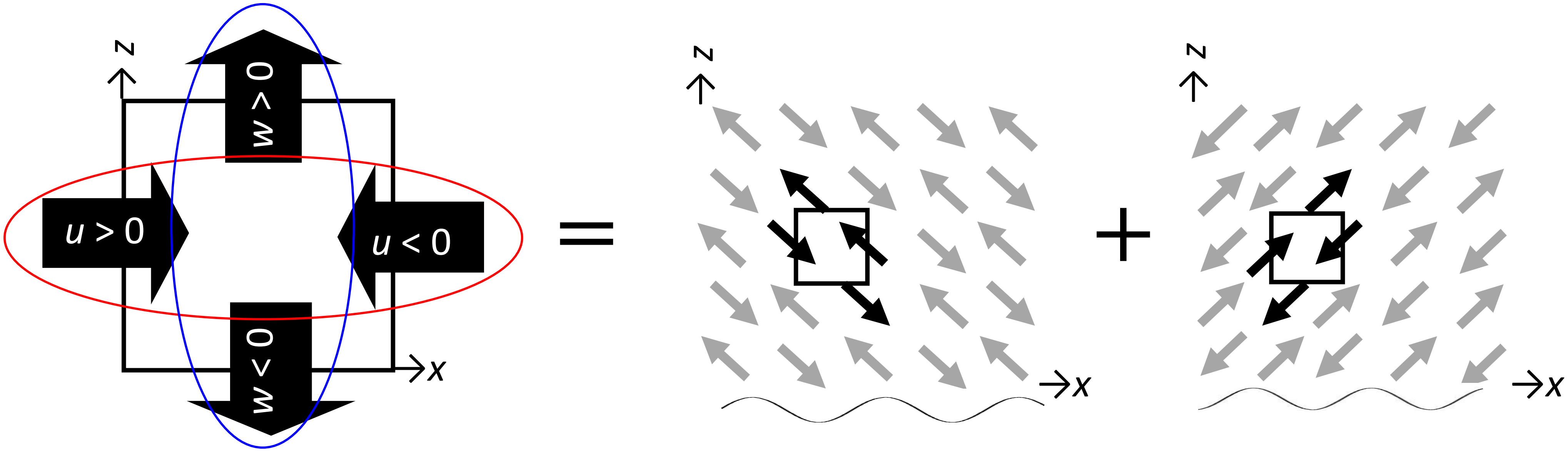
- "function": $y = f(x)$
- π, e, i (1748)
- Trigonometric expansion (→ Fourier series)
- Newton's 2nd law ("equation of motion") (1736)

$$F = m\,a$$

- "Continuity equation" for incompressible inviscid fluid (1757)

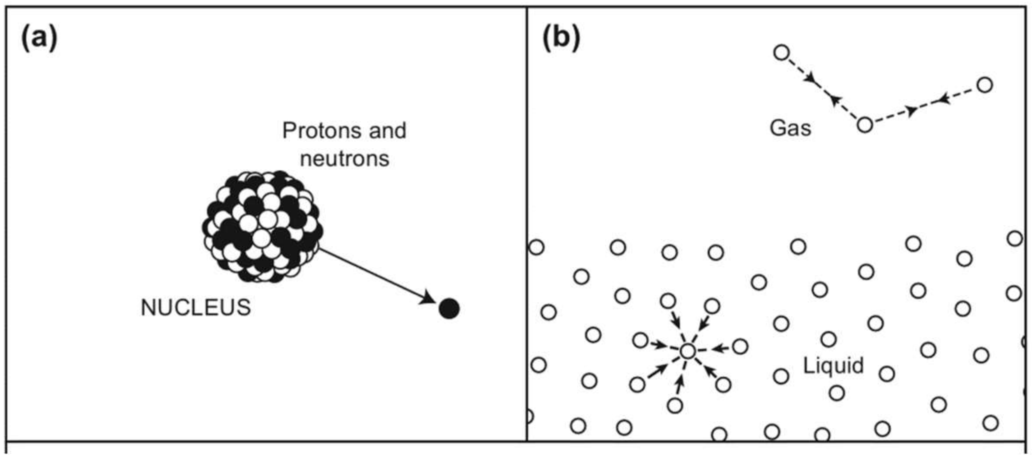
$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0.$$

< 0 > 0

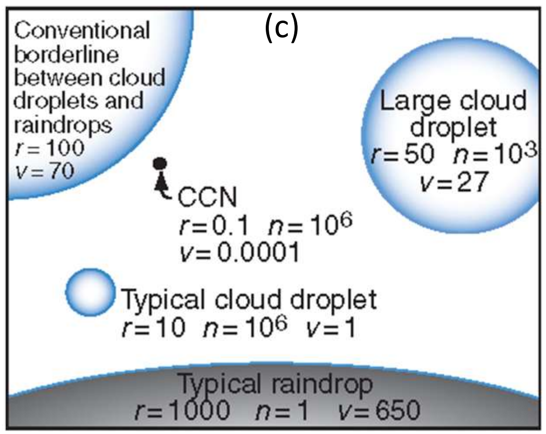


Momentum conservation and Forces (interactions) working in the nature

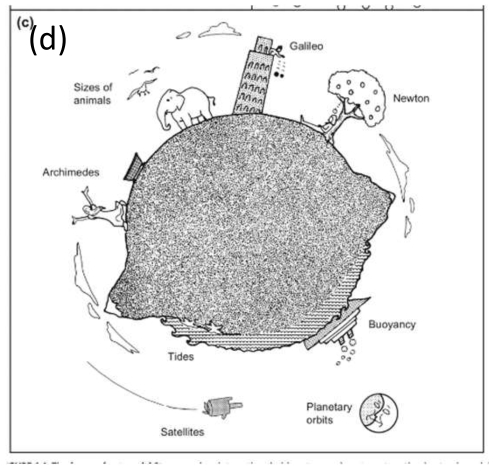
- **Intermolecular (electromagnetic) force**
  → saturation → homogeneous nucleation
  - Kelvin's curvature effect
    → supercooled tiny droplet
  - Raoult's solute effect
    → vapor pressure / boiled point depression
  - Henry-Dalton's partial pressure law
    for a mixed gas
- Unsaturated surface of a droplet
  → molecular diffusion → evaporation
- Condensation at a solid surface
  (heterogeneous nucleation) → large droplet
- Planetary **gravitational force**
  → Density (hydrostatic) stratification → Ocean
  - Photochemical / volcanic water vapor production
  - Gravitational separation / photodissociation
    → Hydrogen escape / oxidation → Ocean loss
- **Precipitation** (coalescence / sublimation) process
  - Gravity, radiation → Equatorial tropopausal "cold trap"
  - Orography / sea-land heat contrast → forced convection
  - Conditional instability → moist convection



Strong/weak nuclear & intermolecular electromagnetic forces
(Israelachvili, 1985, 1992)



Cloud/precipitation processes
(Wallace & Hobbs, 1972, 2006)

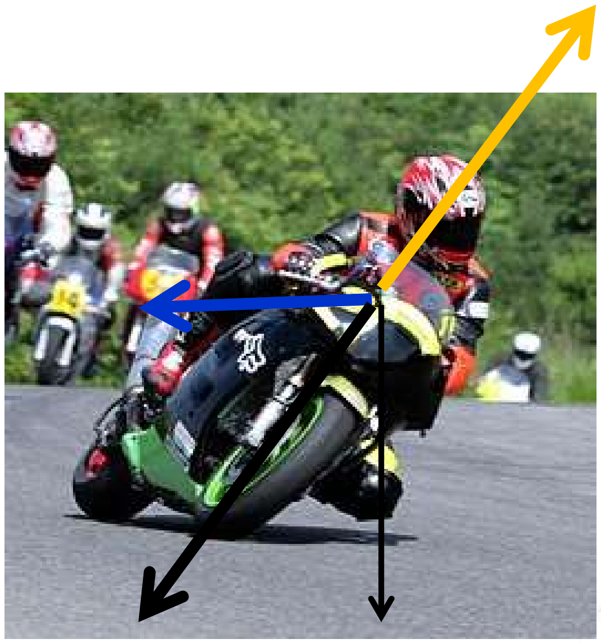


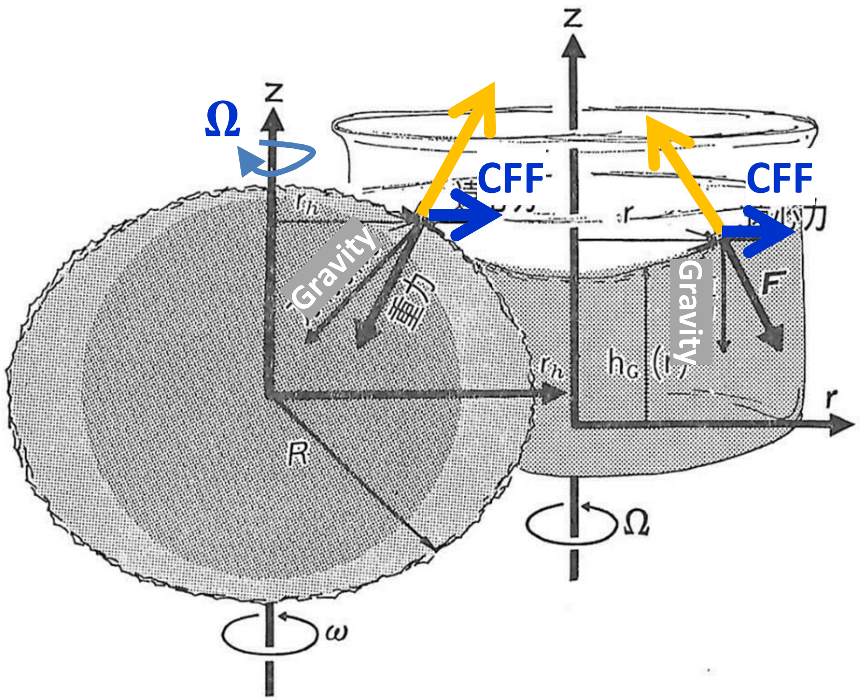
Planetary gravitation
(Israelachvili, 1985, 1992)

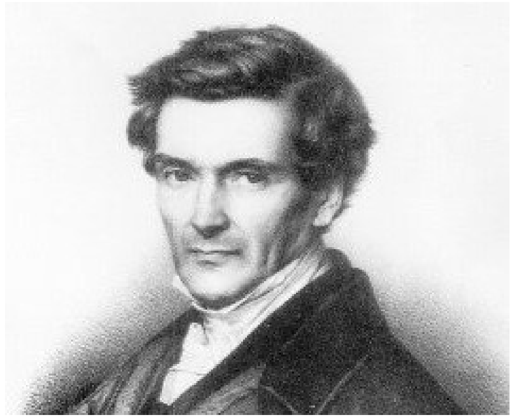
Centrifugal and Coriolis Forces

Centrifugal force $= mr\Omega^2 = \frac{mV^2}{r}$ (m: mass, r: rotation radius, Ω: angular velocity, $V = r\,\Omega$: moving speed)

must be balanced with … pressure gradient etc.







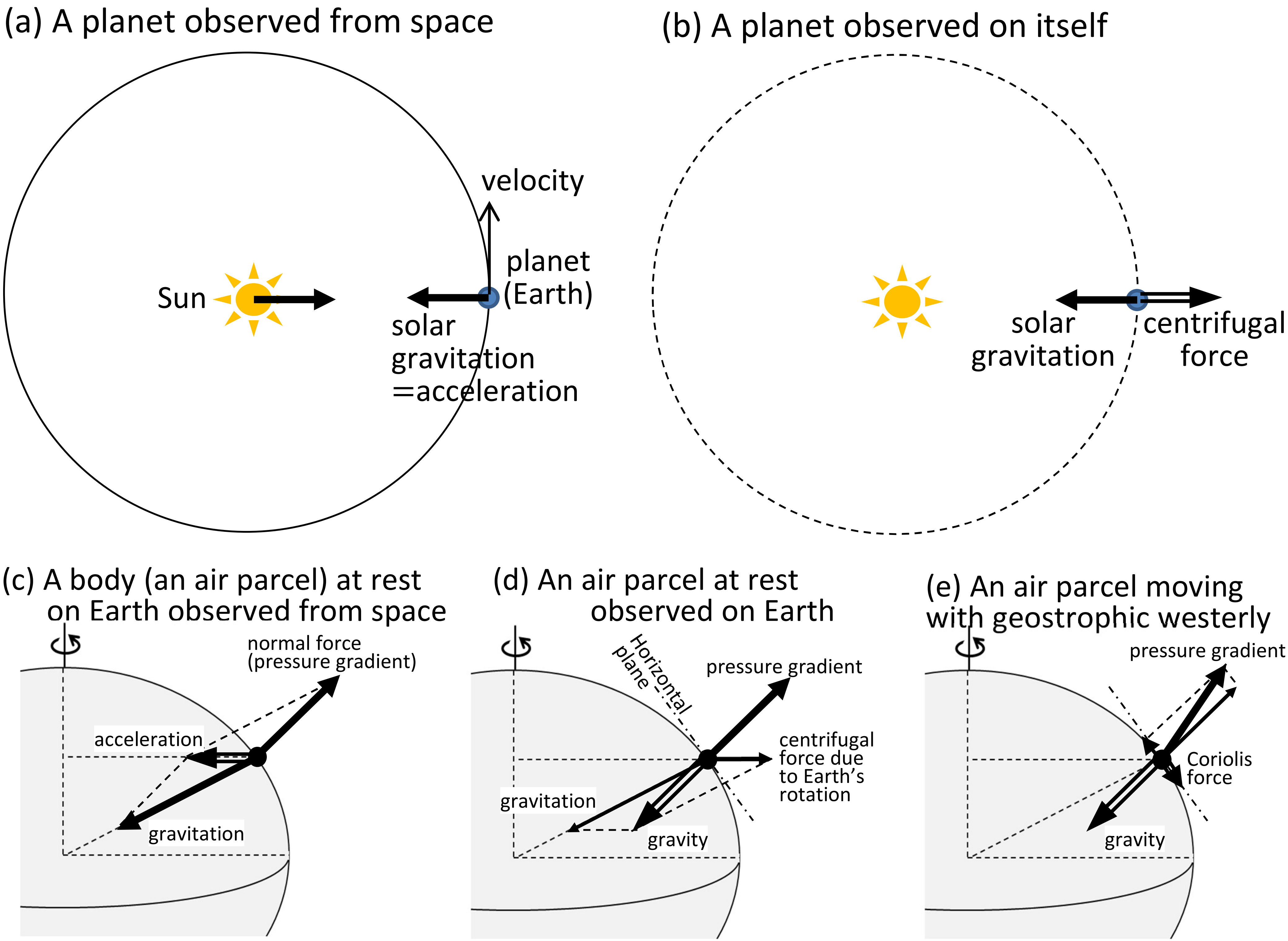
Gaspard-Gustave Coriolis (1792 – 1843)

If the body moves eastward by u (relative to the earth) at latitude φ,

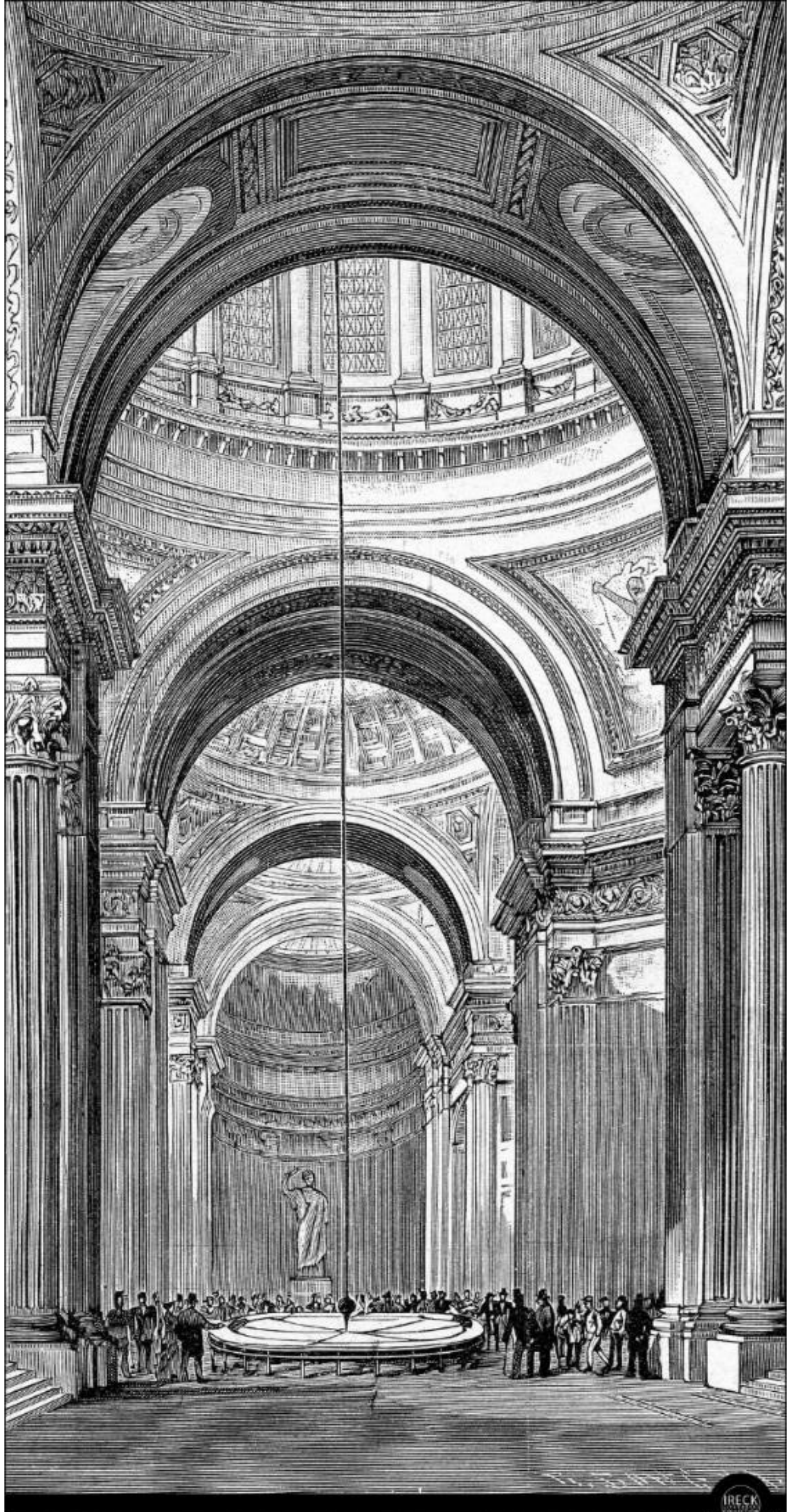
Horizontal component of “total” centrifugal force $= m\,r\cos\varphi\left(\Omega + \frac{u}{r\cos}\right)^2 \sin\varphi$

$$= m\left(\frac{1}{2}r\Omega^2\sin 2\varphi + u\cdot\boxed{2\Omega\sin\varphi} + \frac{u^2}{r}\tan\varphi\right)$$

Coriolis parameter f



(a) A planet observed from space
velocity
planet
(Earth)
Sun
solar
gravitation
=acceleration
(b) A planet observed on itself
solar
gravitation
centrifugal
force
(c) A body (an air parcel) at rest
on Earth observed from space
normal force
(pressure gradient)
acceleration
gravitation
(d) An air parcel at rest
observed on Earth
Horizontal
plane
pressure gradient
centrifugal
force due
to Earth's
rotation
gravitation
gravity
(e) An air parcel moving
with geostrophic westerly
pressure gradient
Coriolis
force
gravity



Foucaults Pendelversuch im Pantheon zu Paris.

Pendulum experiment (at Paris in Feb 1851) by Jean Bernard Léon Foucault (1819 - 1868)

https://www.flickr.com/photos/94791180@N06/13965137961

Earth's rotation around local "vertical" line:

$$\text{Period} = \frac{2\pi}{\Omega \sin\varphi} = \frac{86164\ \text{s}}{\sin\varphi} = \frac{23\text{h}\ 56\text{min}\ 04\text{s}}{\sin\varphi} = \frac{23.934\ \text{h}}{\sin\varphi}$$

$$\frac{\text{Oscillation plane shift}}{1\ \text{hour}} = \frac{360^{\circ}}{\text{Period [h]}} = 15.041^{\circ}\ \sin\varphi$$

(Earth: anticlockwise → Oscillation plane seen clockwise)

North pole ($\varphi = +\,90^{\circ}$):

Period = 23.934 h, Oscillation shift = 15.041°/h

Paris ($\varphi = +\,48^{\circ}51'$):

Period = 31.785 h, Oscillation shift = 11.326°/h

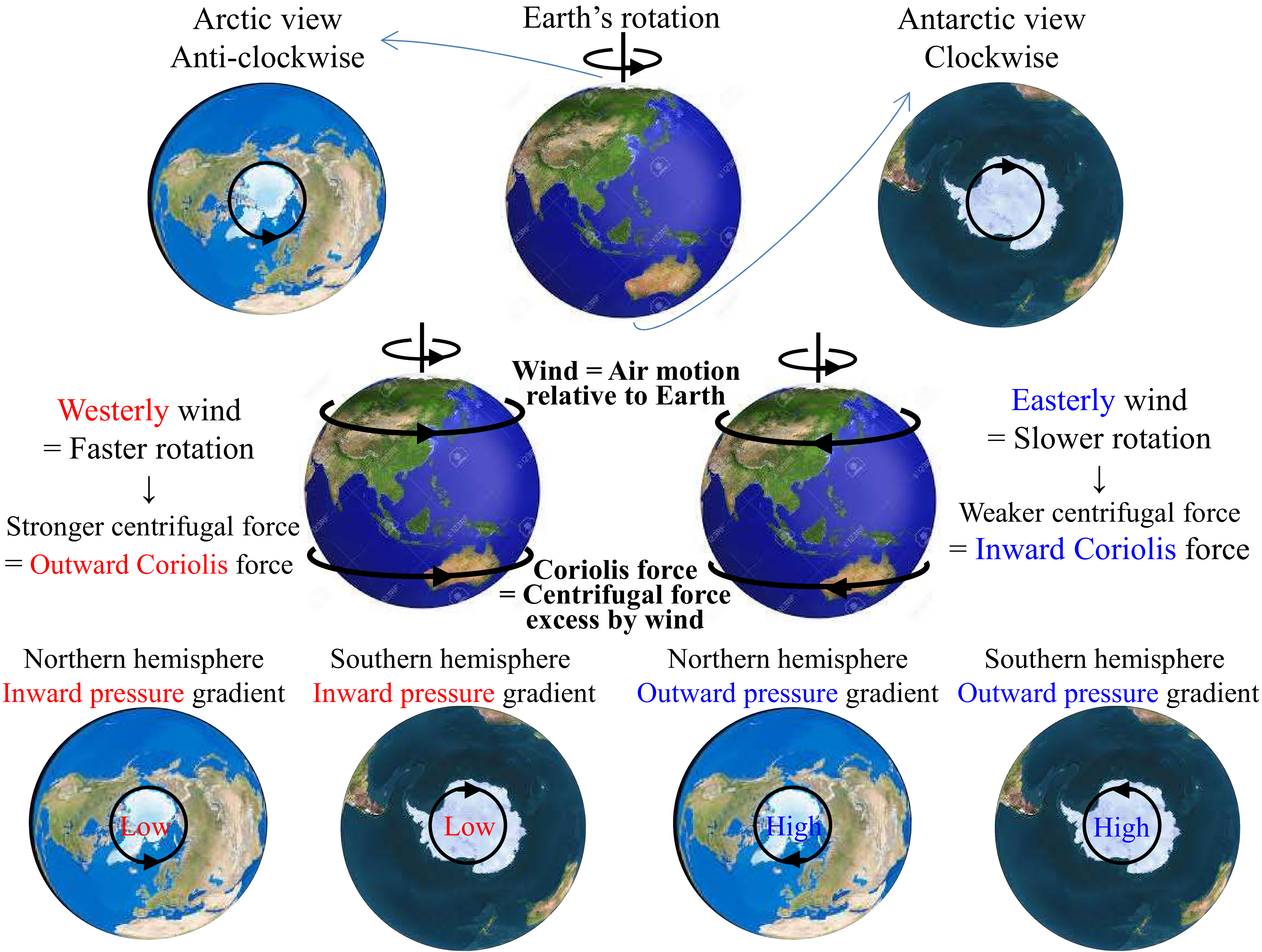
Colombo ($\varphi = +\,6^{\circ}56'$): (1st low lat case by Lamprey & Shaw, Sep.1851)

Period = 198.27 h, Oscillation shift = 1.816°/h

Equator ($\varphi = 0^{\circ}$): Period = ∞, Hourly oscillation shift = 0

South pole ($\varphi = -\,90^{\circ}$): Inverse (seen anti-clockwise) rotation

Period = − 23.934 h, Oscillation shift = − 15.041°/h



Arctic view
Anti-clockwise
Earth's rotation
Antarctic view
Clockwise
Wind = Air motion relative to Earth
Westerly wind
= Faster rotation
↓
Stronger centrifugal force
= Outward Coriolis force
Easterly wind
= Slower rotation
↓
Weaker centrifugal force
= Inward Coriolis force
Coriolis force
= Centrifugal force excess by wind
Northern hemisphere
Inward pressure gradient
Southern hemisphere
Inward pressure gradient
Northern hemisphere
Outward pressure gradient
Southern hemisphere
Outward pressure gradient
Low
Low
High
High

Energy conservation and thermodynamics

- Mechanics for a moving point mass:

  Momentum = mass × velocity

  Kinetic energy = (1/2) mass × $(\text{velocity})^2$

  Gravity potential energy = g mass × vertical displacement

- Equation of state for gas:

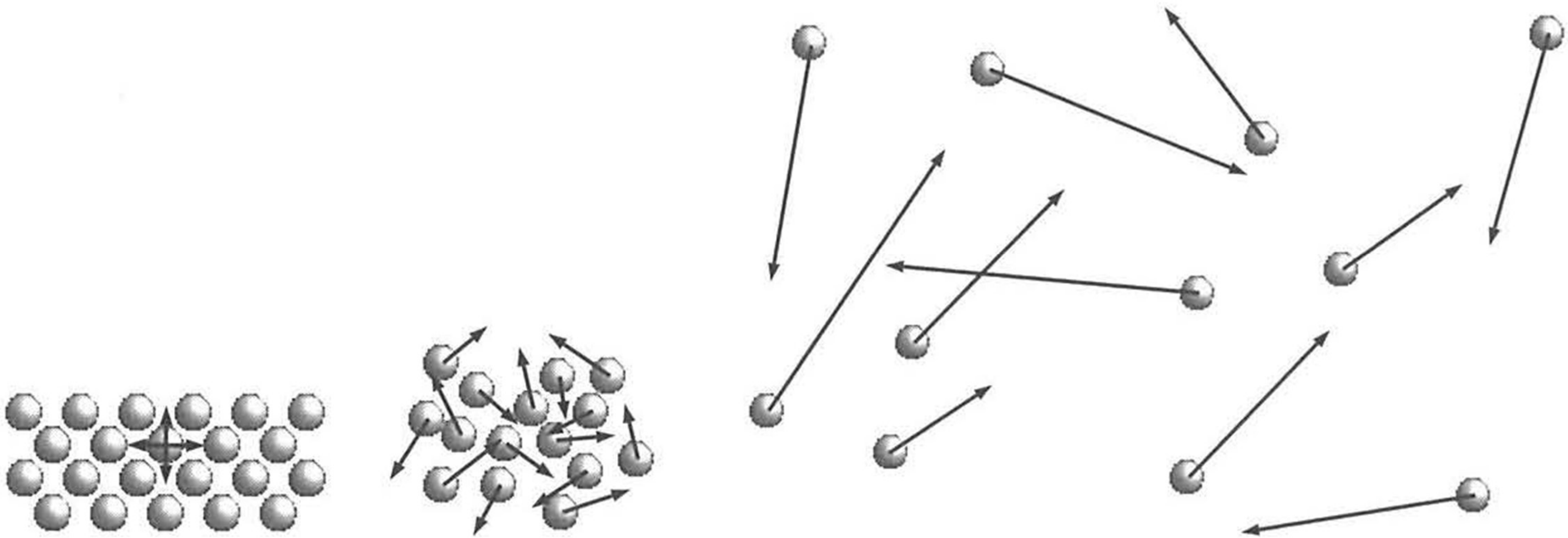
  Pressure ∝ density × temperature

(atmosphere) (ocean,…) (earth)

- Phase of matter: Gas, Liquid, Solid

- Continuum: **Fluid**, Plastic, Elastic, Rigid

Solid (Ice) Liquid (Water) Gas (Vapor)



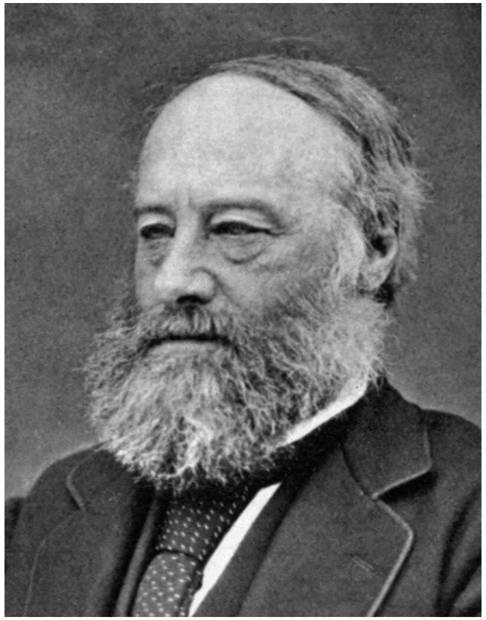
Julius Robert von Mayer
(1814 – 1878)

http://www.kumc.edu
/dc/pc/mayer.jpg

German scientist cruised in 1840 to East Jawa as a Dutch ship doctor, and noticed a concept called energy at present as exchangeable quantity between motion and heat. After returning to Germany in 1841, he submitted a paper to a journal of physics, but rejected. In 1842 his paper was accepted by a journal of chemistry, but was not so highly evaluated. In 1845 his second paper was rejected even by the chemical journal. After that he never submitted any papers to journals but published them by himself. In 1850 he became a farmer until his death.

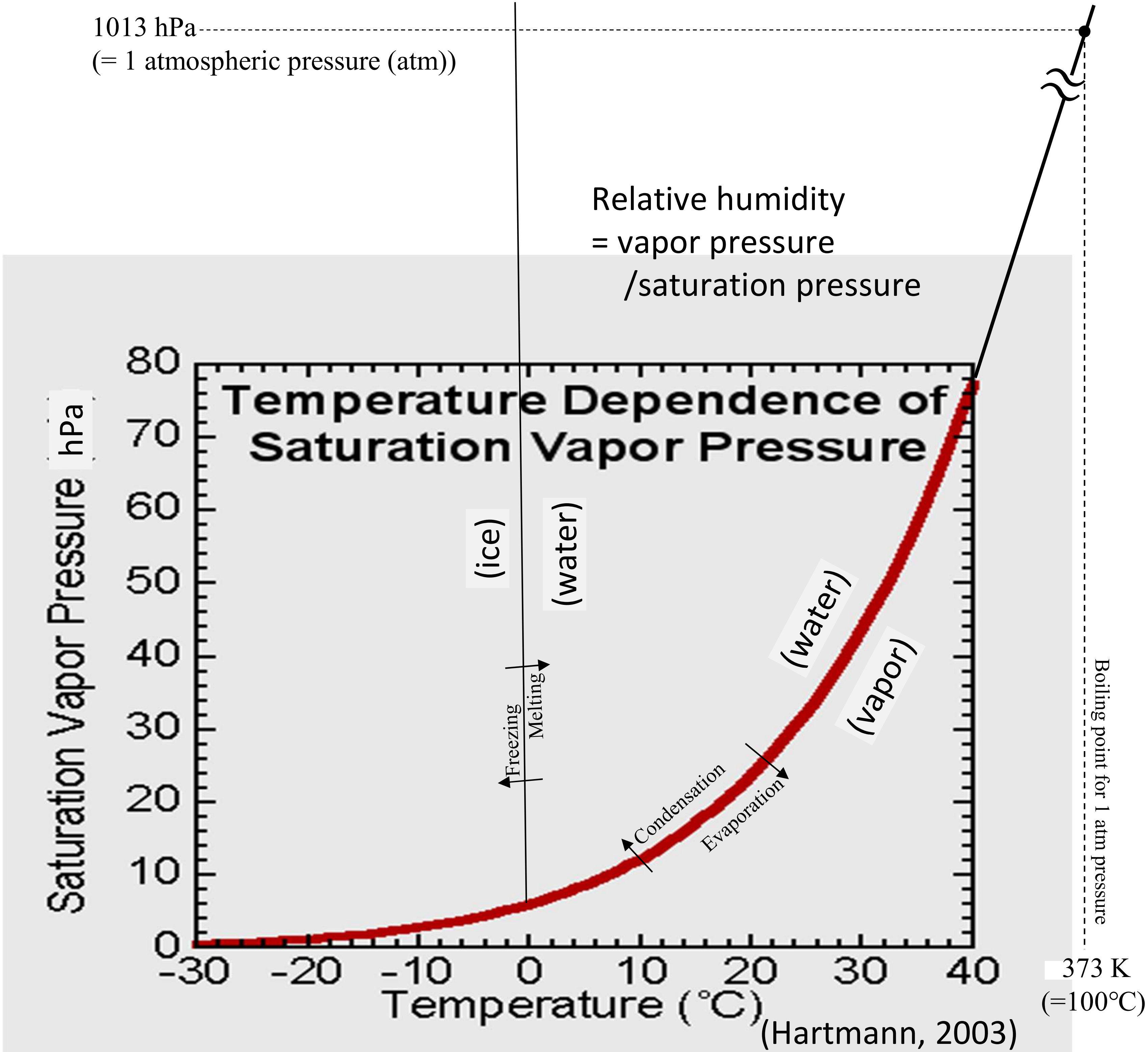
In 1854 von Helmholz recognized that Mayer was the first person discovering the energy.

James Prescott Joule
(1818 – 1889)



(Roscoe, 1906;
https://en.wikipedia.org/
wiki/James_Prescott_Jou
le)

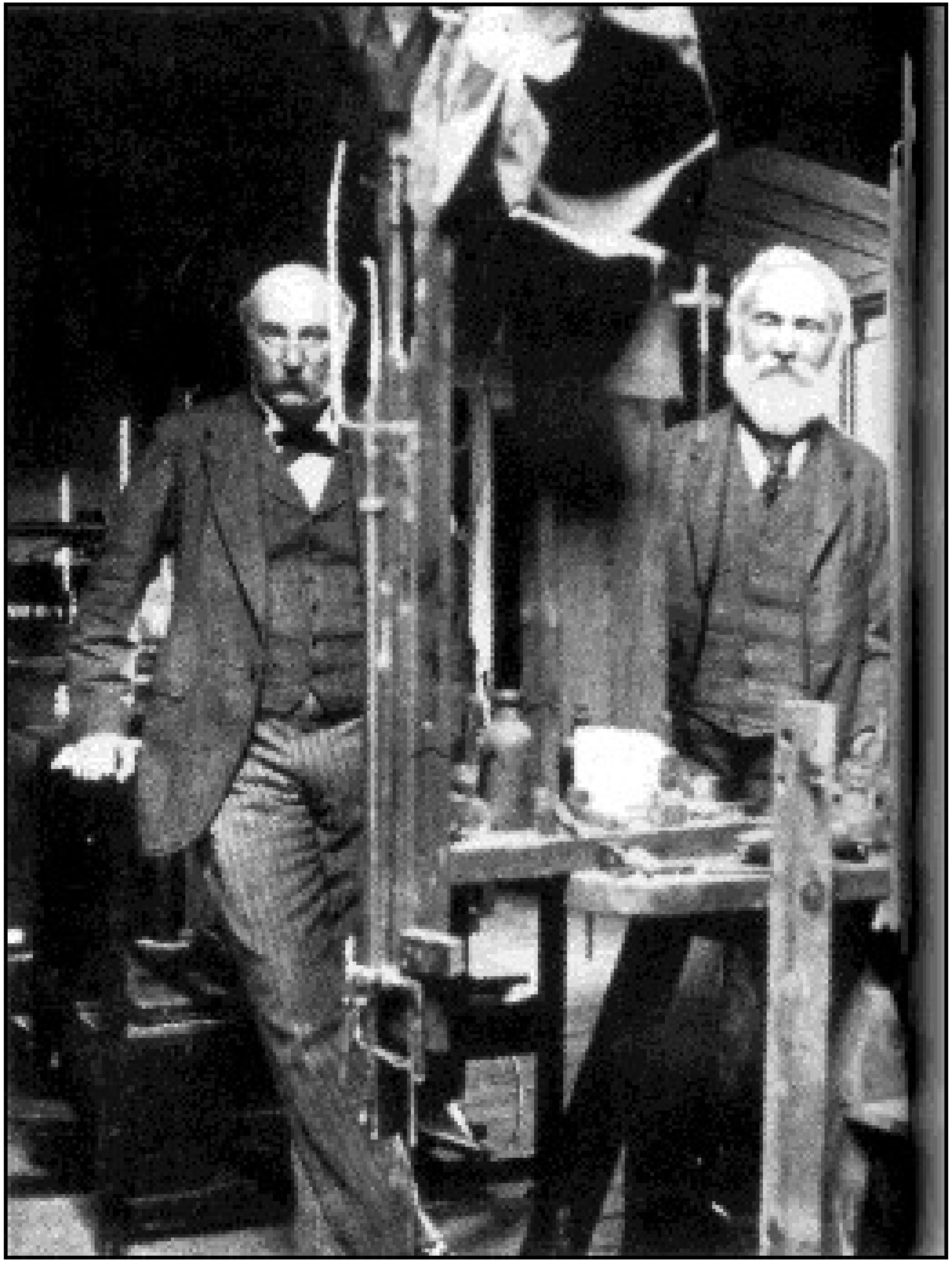
English brewer studied physics without any post at university or institute. He discovered the Joule's law and the mechanical equivalent of heat in early 1840s.



1013 hPa
(= 1 atmospheric pressure (atm))
Relative humidity
= vapor pressure
/saturation pressure
Temperature Dependence of
Saturation Vapor Pressure
Saturation Vapor Pressure hPa
80
70
60
50
40
30
20
10
0
-30
-20
-10
0
10
20
30
40
Temperature (°C)
(ice)
(water)
Freezing
Melting
(water)
(vapor)
Condensation
Evaporation
Boiling point for 1 atm pressure
373 K
(=100°C)
(Hartmann, 2003)

Lord Rayleigh
John William Strutt

(1842-1919)



Lord Kelvin
William Thomson
(1824- 1907)

Why sky is blue?

Shorter wavelength (violet)

=> Particle (Mie) refraction

Blue color light

=> Molecule (Rayleigh) refraction

Longer wavelength (green, yellow, red)

=> Moving straight (sunrise/sunset)

Rainbow by liquid droplets

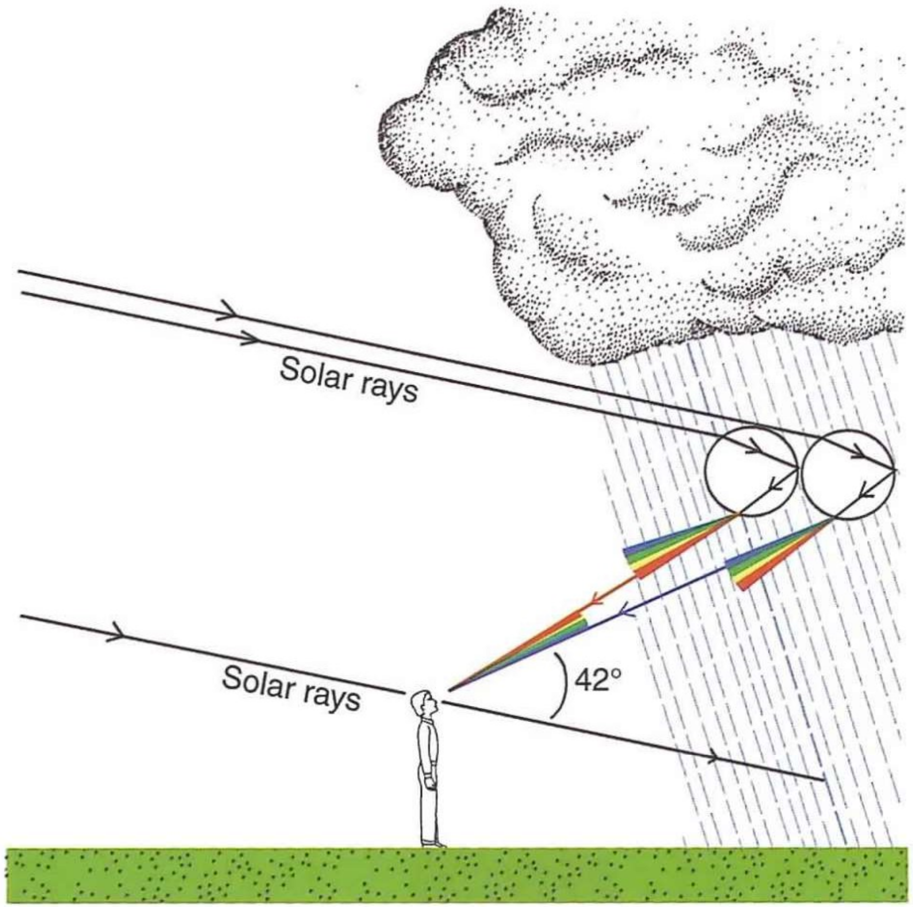
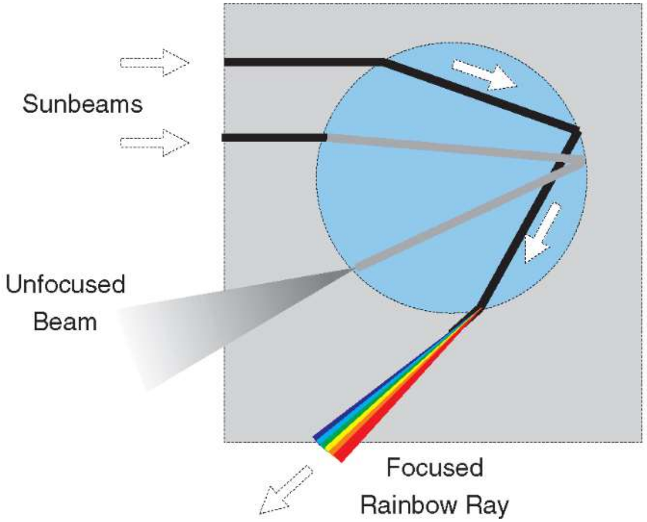


Fig. 4.16 A solar ray is refracted and internally reflected (once) and refracted again by a raindrop to form the (primary) rainbow. The rainbow ray is the brightest and has the smallest angle of deviation of all the rays that encounter raindrop undergo these optical processes. Like a prism, refraction by the raindrop disperses visible light into its component colors forming the rainbow color band. The secondary rainbow is produced by double reflection within raindrops, which appears about 8 degrees above the primary rainbow, with the order of the colors reversed. The rainbow is a mosaic produced by passage of light through the circular cross section of myriad raindrops. (Wallace & Hobbs)



(Gedzelman)

Figure 5 The primary rainbow is produced by passage of light through the circular cross-section of a spherical raindrop. The rainbow ray is the most focused and least deflected light

Fig. 4.15 Primary rainbow with a weaker secondary rainbow above it and supernumerary bows below it. [Photograph courtesy of Joanna Gurstelle.] (Wallace & Hobbs)

Halo
by ice crystals

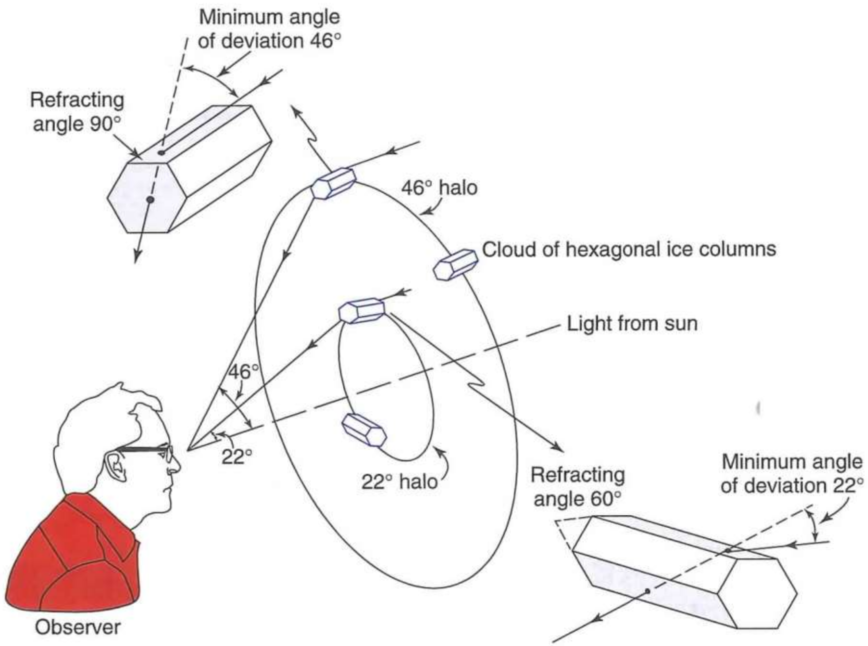


Fig. 4.18 Refraction of light in hexagonal ice crystals to produce the 22° and 46° haloes. (Wallace & Hobbs)

South Pole halo complex, 2 January 1990. (Photograph courtesy of Walter Tape.)

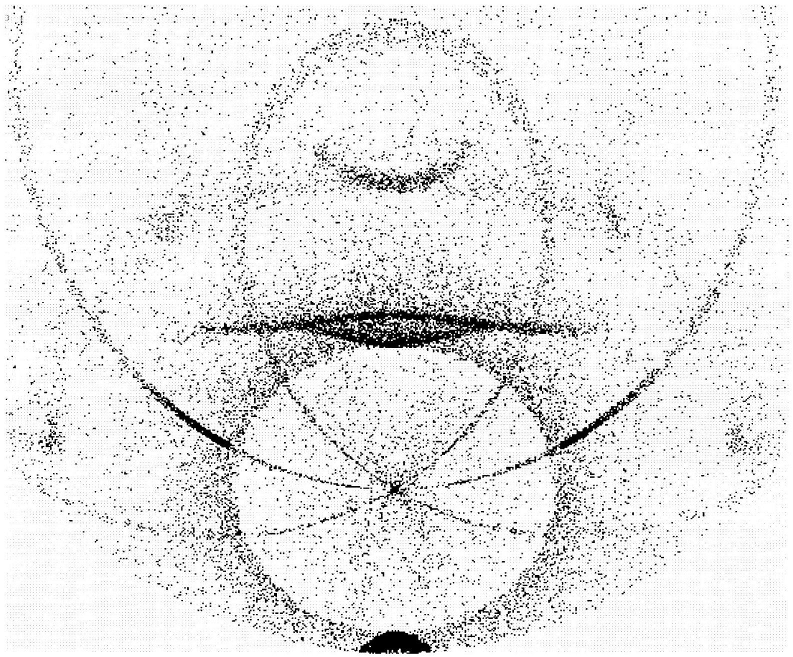
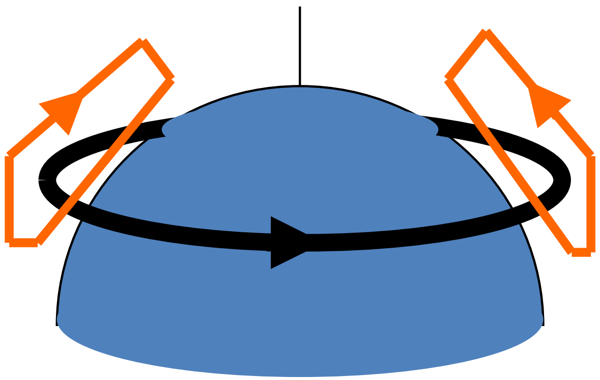
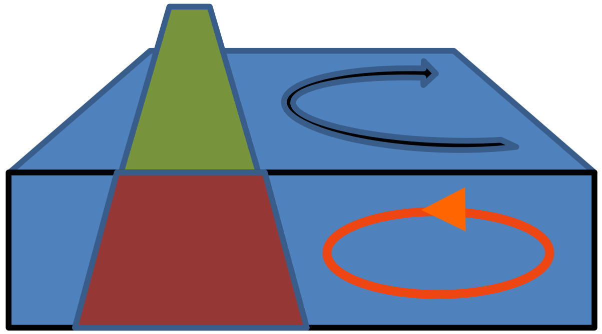
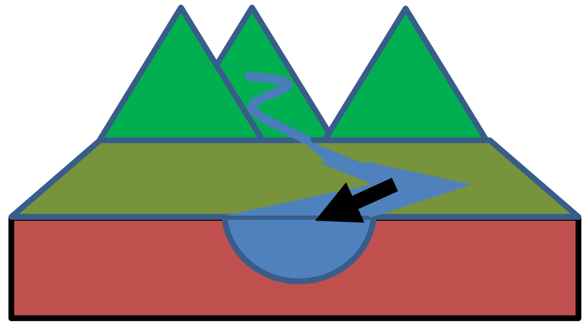


Figure 9 Computer simulation of a complex halo display. Four types of crystals are included. Thick plates falling randomly produce the 22° halo. Thin plates falling almost horizontally produce the parhelia, the parhelic circle, and the circumzenithal arc. Long pencils falling almost horizontally but with random orientation of rectangular sides produce the upper tangent arc to the 22° halo, and the faint supralateral and infralateral tangent arcs to the 46° halo. Long pencils falling almost horizontally with one rectangular face almost horizontal produce the Tape arcs and the Parry arc.

(Gedzelman)

Various fluid flows in the Earth System

[(i) Momentum and (ii) continuity eqs. are common for any cases]

Global atmosphere (Meteorology)	Ocean (Oceanography)	River (Hydrology)
		
Compressible [+thermodynamics (iii)(iv)]	Incompressible [+thermal expansion]	Incompressible [+Level/stream change]
Almost closed	Almost closed	Opened
Zonal dominant	Horizontal dominant	Almost one-dimensional
Moisture effect [(v)]	Salinity effect	Complex boundary
Almost free	Coastal effect	