2. Conservation laws and basic equations

Actual matter = Σ molecule

- Density = Σ molecular mass / unit volume
  = molecular mass × number / unit volume

- Pressure = Σ molecular momentum / unit time / unit area
  = molecular force / unit area

- Temperature = Σ molecular kinetic energy
  / molecular number / Boltzmann constant
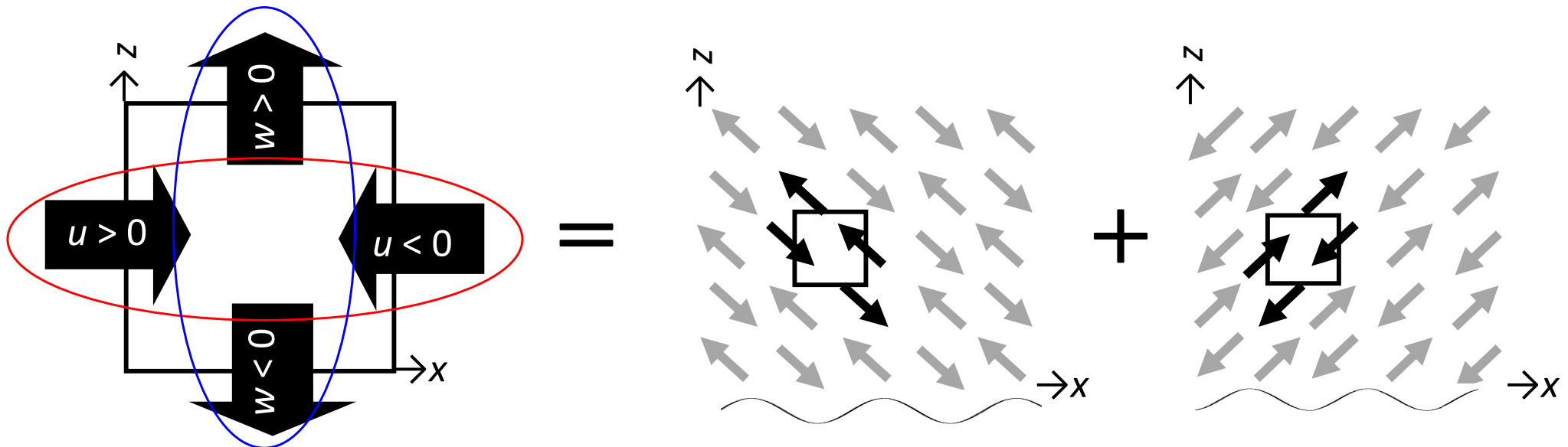
Mass conservation and fluid continuity

Leonhard Euler (1707～1783)

- "function": $y = f(x)$

- π, e, i (1748)

- Trigonometric expansion ($\rightarrow$ Fourier series)

- Newton's 2nd law ("equation of motion") (1736)

$$F = m\,a$$

- "Continuity equation" for incompressible inviscid fluid (1757)

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0.$$

$< 0 \qquad > 0$

Momentum conservation and Forces (interactions) working in the nature

- **Intermolecular (electromagnetic) force**
  → saturation → homogeneous nucleation
  - Kelvin's curvature effect
             → supercooled tiny droplet
  - Raoult's solute effect
             → vapor pressure / boiled point depression
  - Henry-Dalton's partial pressure law
                            for a mixed gas
- Unsaturated surface of a droplet
 → molecular diffusion → evaporation

- Condensation at a solid surface
             (heterogeneous nucleation) → large droplet
- Planetary **gravitational force**
             → Density (hydrostatic) stratification → Ocean
  - Photochemical / volcanic water vapor production
  - Gravitational separation / photodissociation
             → Hydrogen escape / oxidation → Ocean loss
- **Precipitation** (coalescence / sublimation) process
  - Gravity, radiation → Equatorial tropopausal "cold trap"
  - Orography / sea-land heat contrast → forced convection
  - Conditional instability → moist convection
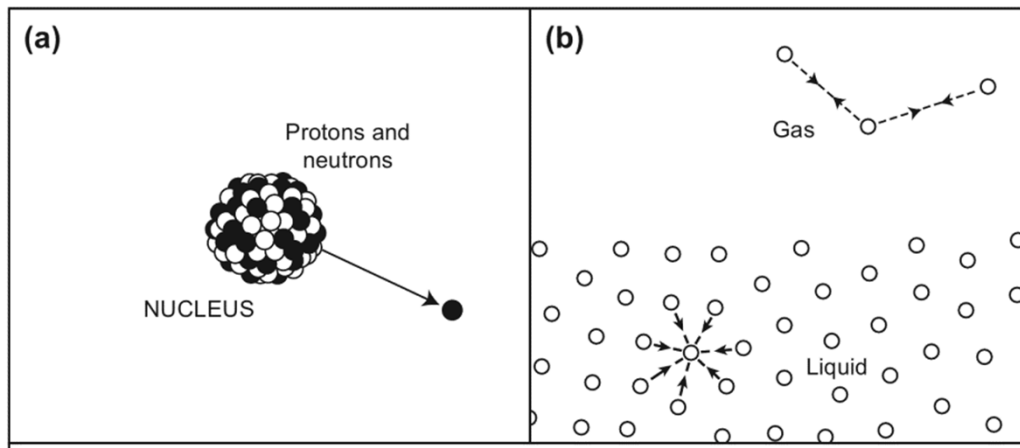
Strong/weak nuclear & intermolecular electromagnetic forces
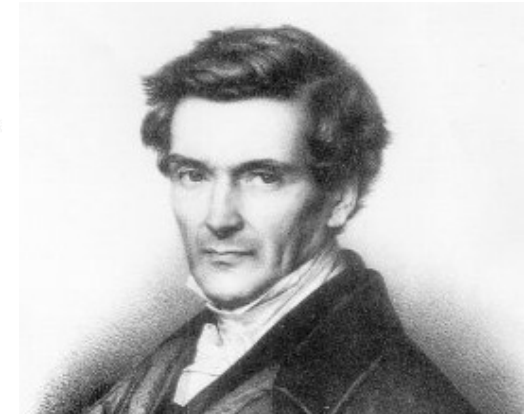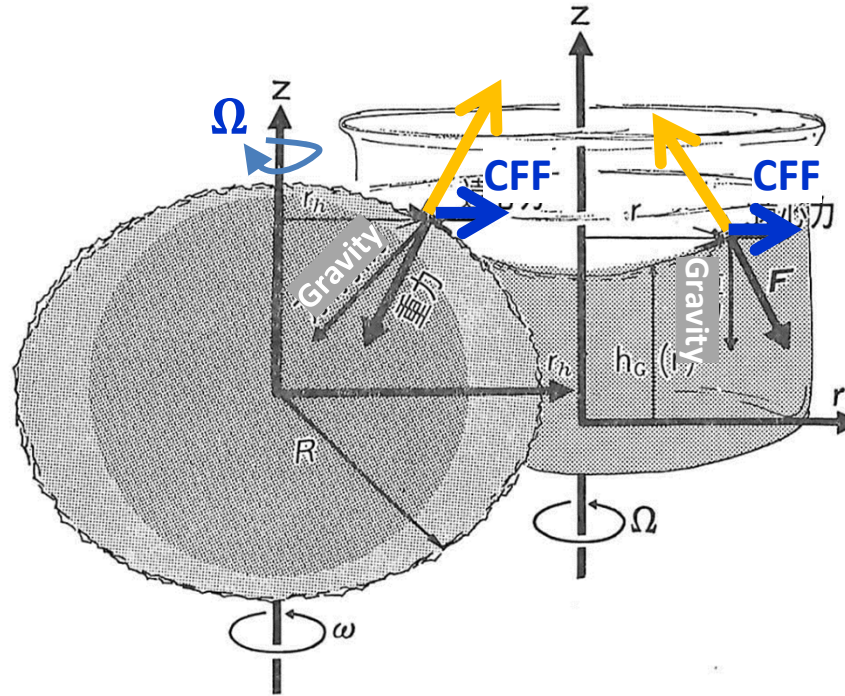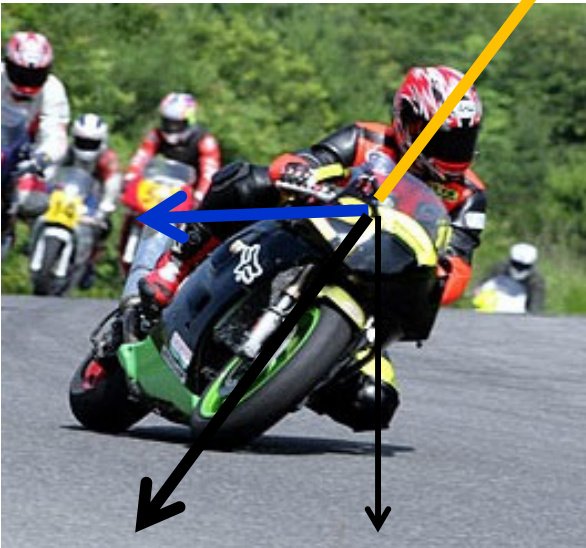(Israelachvili, 1985, 1992)
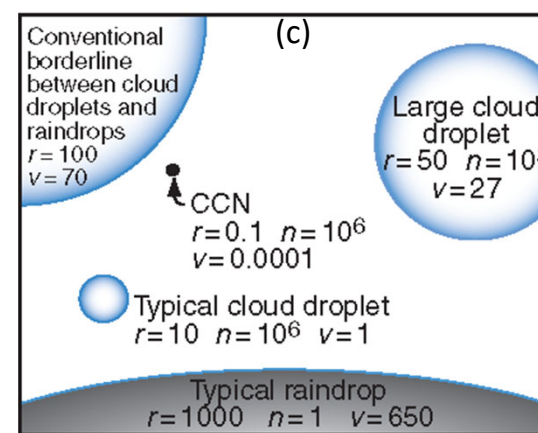
Cloud/precipitation processes
(Wallace & Hobbs, 1972, 2006)
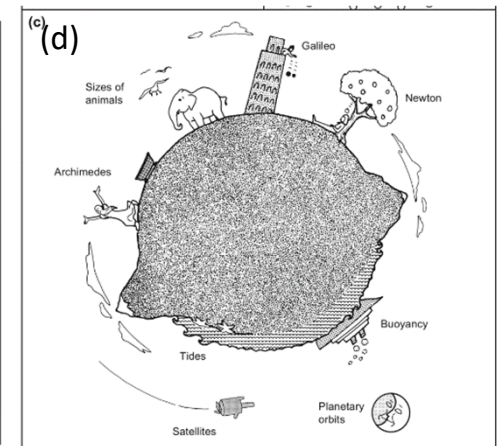
Planetary gravitation
(Israelachvili, 1985, 1992)

Centrifugal and Coriolis Forces

Centrifugal force $= mr\Omega^2 = \dfrac{mV^2}{r}$ (m: mass, r: rotation radius, Ω: angular velocity,

must be balanced with … pressure gradient etc.

$$V = r\,\Omega : \text{ moving speed)}$$

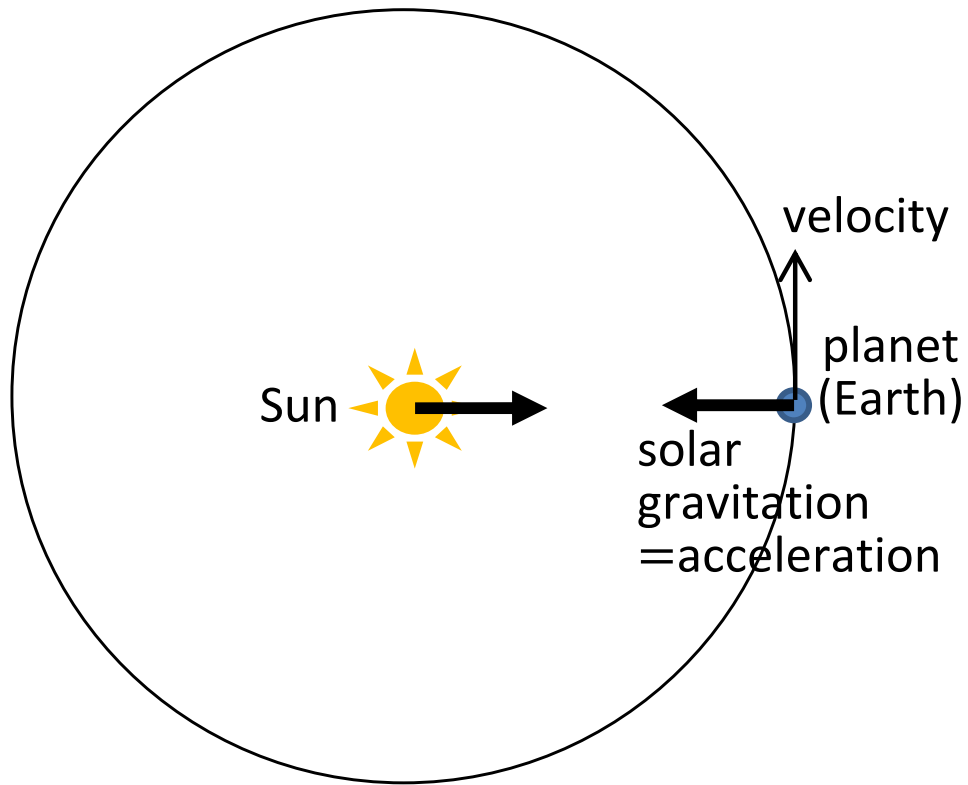**Gaspard-Gustave Coriolis**
(1792 − 1843)

If the body moves eastward by u (relative to the earth) at latitude φ,

Horizontal component of "total" centrifugal force $= m\,r\cos\varphi\left(\Omega + \dfrac{u}{r\cos\varphi}\right)^2 \sin\varphi$
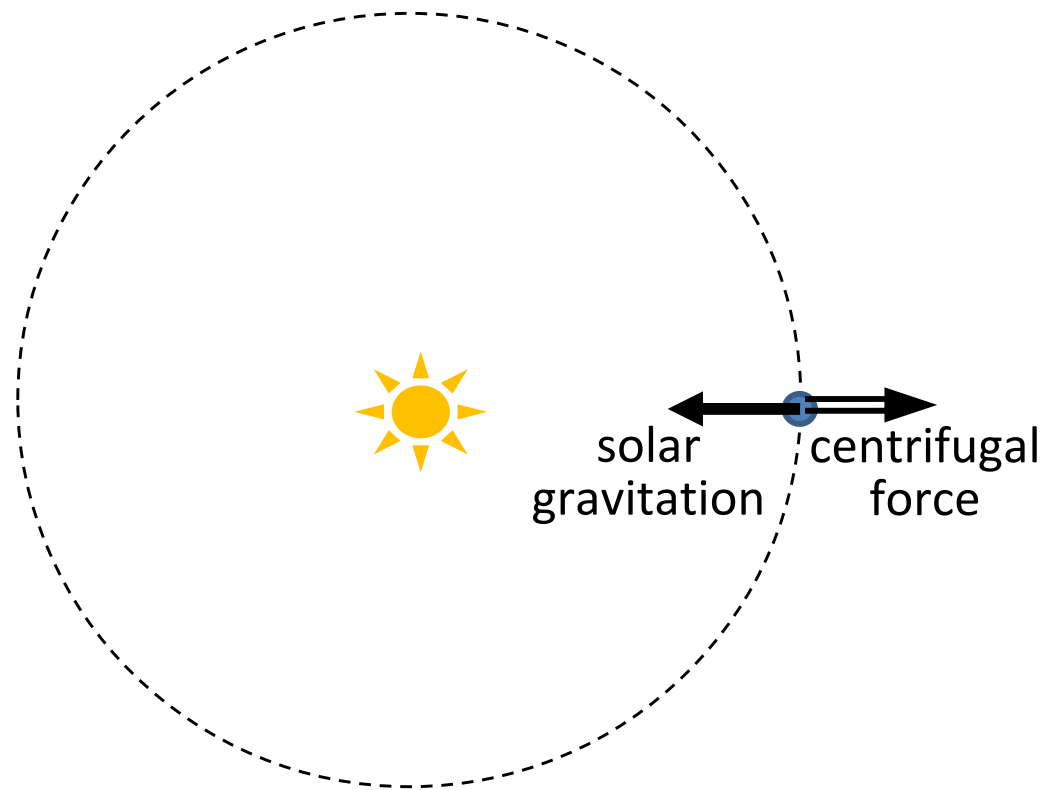
$$= m\left(\frac{1}{2}r\Omega^2 \sin 2\varphi + u \cdot \boxed{2\Omega\sin\varphi} + \frac{u^2}{r}\tan\varphi\right)$$
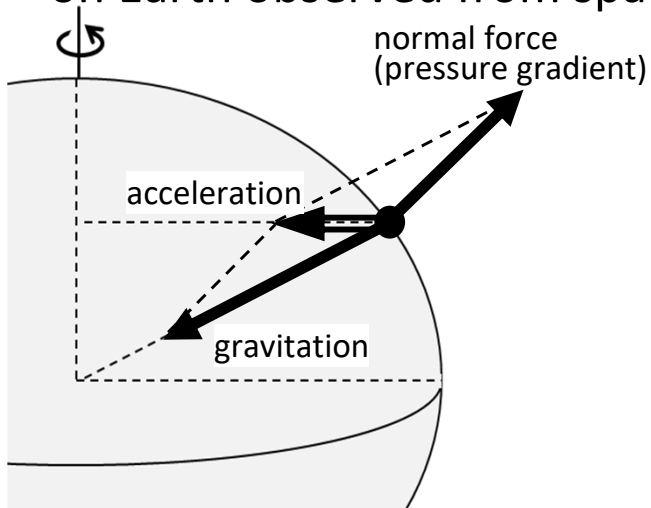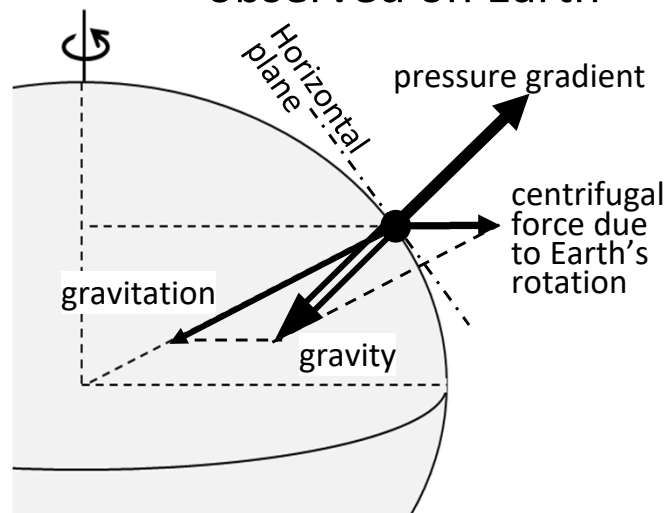
Coriolis parameter f

(a) A planet observed from space

velocity

Sun

planet (Earth)

solar gravitation =acceleration

(b) A planet observed on itself

solar gravitation

centrifugal force

(c) A body (an air parcel) at rest on Earth observed from space

normal force (pressure gradient)

acceleration

gravitation

(d) An air parcel at rest observed on Earth

Horizontal plane

pressure gradient

centrifugal force due to Earth's rotation

gravitation

gravity

(e) An air parcel moving with geostrophic westerly

pressure gradient

Coriolis force

gravity

Foucaults Pendelversuch im Pantheon zu Paris.

Pendulum experiment (at Paris in Feb 1851) by Jean Bernard Léon Foucault (1819 - 1868)

https://www.flickr.com/photos/94791180@N06/13965137961

Earth's rotation around local "vertical" line:

$$\text{Period} = \frac{2\pi}{\Omega \sin \varphi} = \frac{86164 \text{ s}}{\sin \varphi} = \frac{23\text{h } 56\text{min } 04\text{s}}{\sin \varphi} = \frac{23.934 \text{ h}}{\sin \varphi}$$

$$\frac{\text{Oscillation plane shift}}{1 \text{ hour}} = \frac{360^\circ}{\text{Period [h]}} = 15.041^\circ \sin \varphi$$

(Earth: <u>anticlockwise</u> → Oscillation plane seen clockwise)

North pole ($\varphi = + 90^\circ$):

    Period = 23.934 h, Oscillation shift = 15.041°/h

Paris ($\varphi = + 48^\circ 51'$):

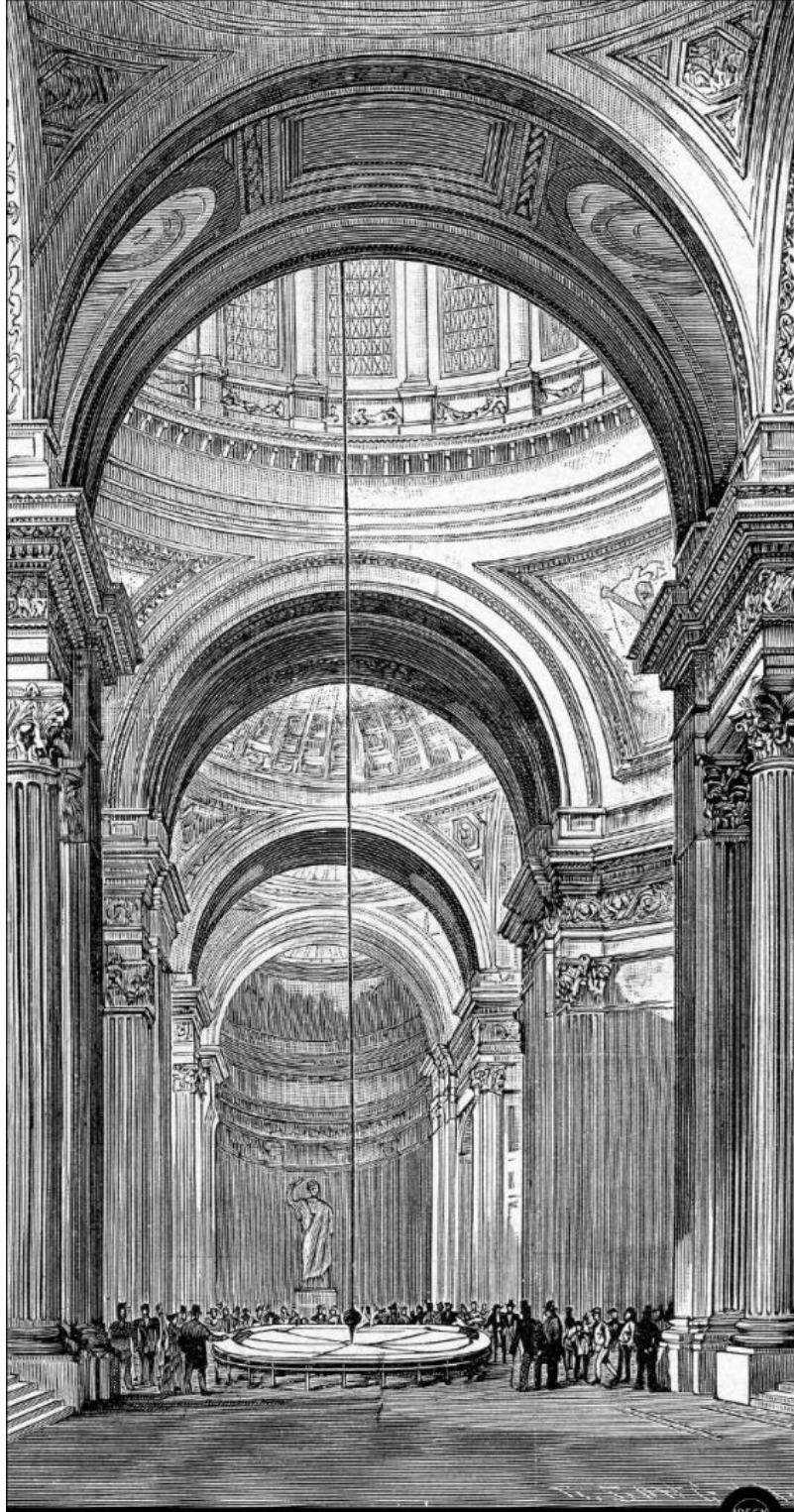   Period = 31.785 h, Oscillation shift = 11.326°/h

Colombo ($\varphi = + 6^\circ 56'$): (1st low lat case by Lamprey & Shaw, Sep.1851)

    Period = 198.27 h, Oscillation shift = 1.816°/h

Equator ($\varphi = 0^\circ$):   Period = ∞, Hourly oscillation shift = 0

South pole ($\varphi = - 90^\circ$): Inverse (seen anti-clockwise) rotation

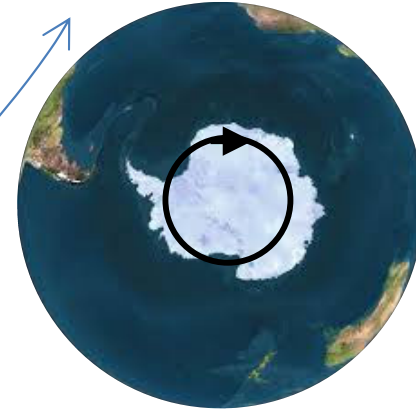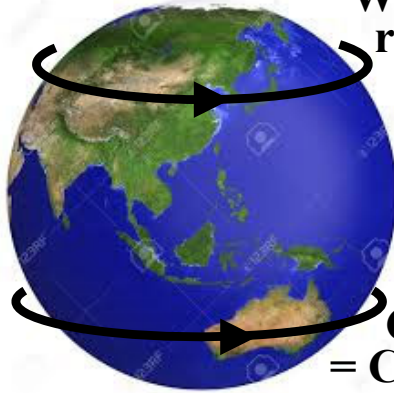   Period = − 23.934 h, Oscillation shift = − 15.041°/h

Arctic view
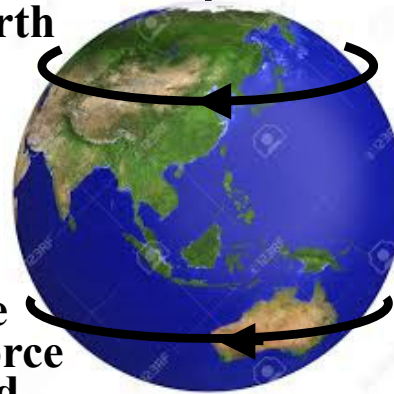Anti-clockwise

Earth's rotation

Antarctic view
Clockwise

Westerly wind
= Faster rotation
↓
Stronger centrifugal force
= Outward Coriolis force
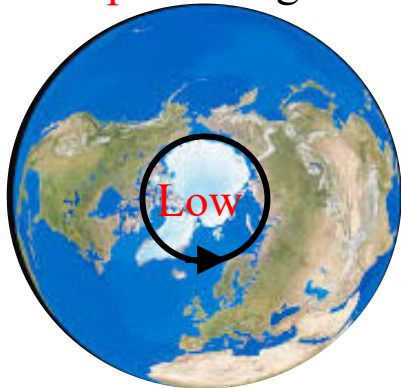
Wind = Air motion
relative to Earth
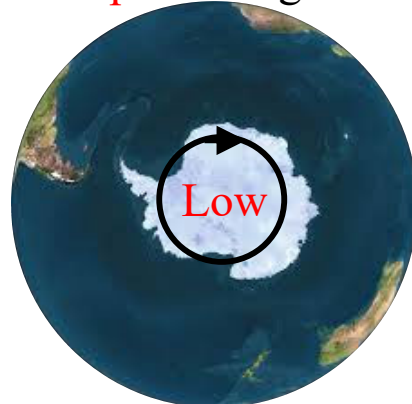
Coriolis force
= Centrifugal force
excess by wind

Easterly wind
= Slower rotation
↓
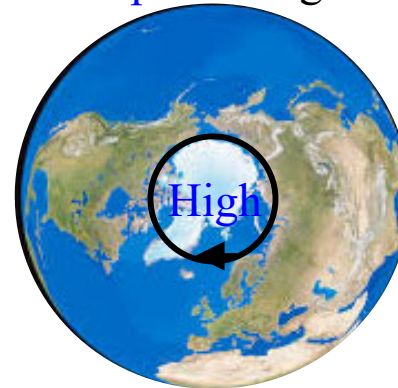Weaker centrifugal force
= Inward Coriolis force

Northern hemisphere
Inward pressure gradient

Southern hemisphere
Inward pressure gradient

Northern hemisphere
Outward pressure gradient

Southern hemisphere
Outward pressure gradient

Low

Low

High

High
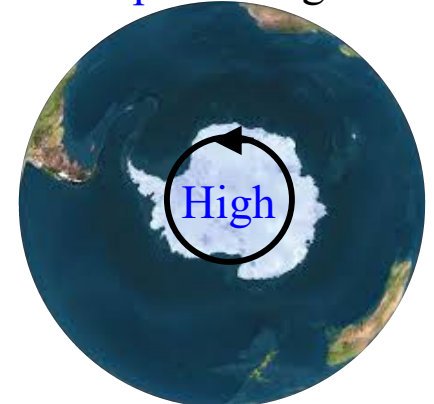
Energy conservation and thermodynamics

- Mechanics for a moving point mass:

  Momentum = mass × velocity

  Kinetic energy = (1/2) mass × (velocity)^2

  Gravity potential energy = g mass × vertical displacement

- Equation of state for gas:

  Pressure ∝ density × temperature ←もっと説明

  (atmosphere) (ocean,…) (earth)

- Phase of matter: Gas, Liquid, Solid

- Continuum: **Fluid**, Plastic, Elastic, Rigid

Julius Robert von Mayer
(1814 – 1878)

http://www.kumc.edu/dc/pc/mayer.jpg

German scientist cruised in 1840 to East Jawa as a Dutch ship doctor, and noticed a concept called energy at present as exchangeable quantity between motion and heat. After returning to Germany in 1841, he submitted a paper to a journal of physics, but rejected. In 1842 his paper was accepted by a journal of chemistry, but was not so highly evaluated. In 1845 his second paper was rejected even by the chemical journal. After that he never submitted any papers to journals but published them by himself. In 1850 he became a farmer until his death.

In 1854 von Helmholz recognized that Mayer was the first person discovering the energy.
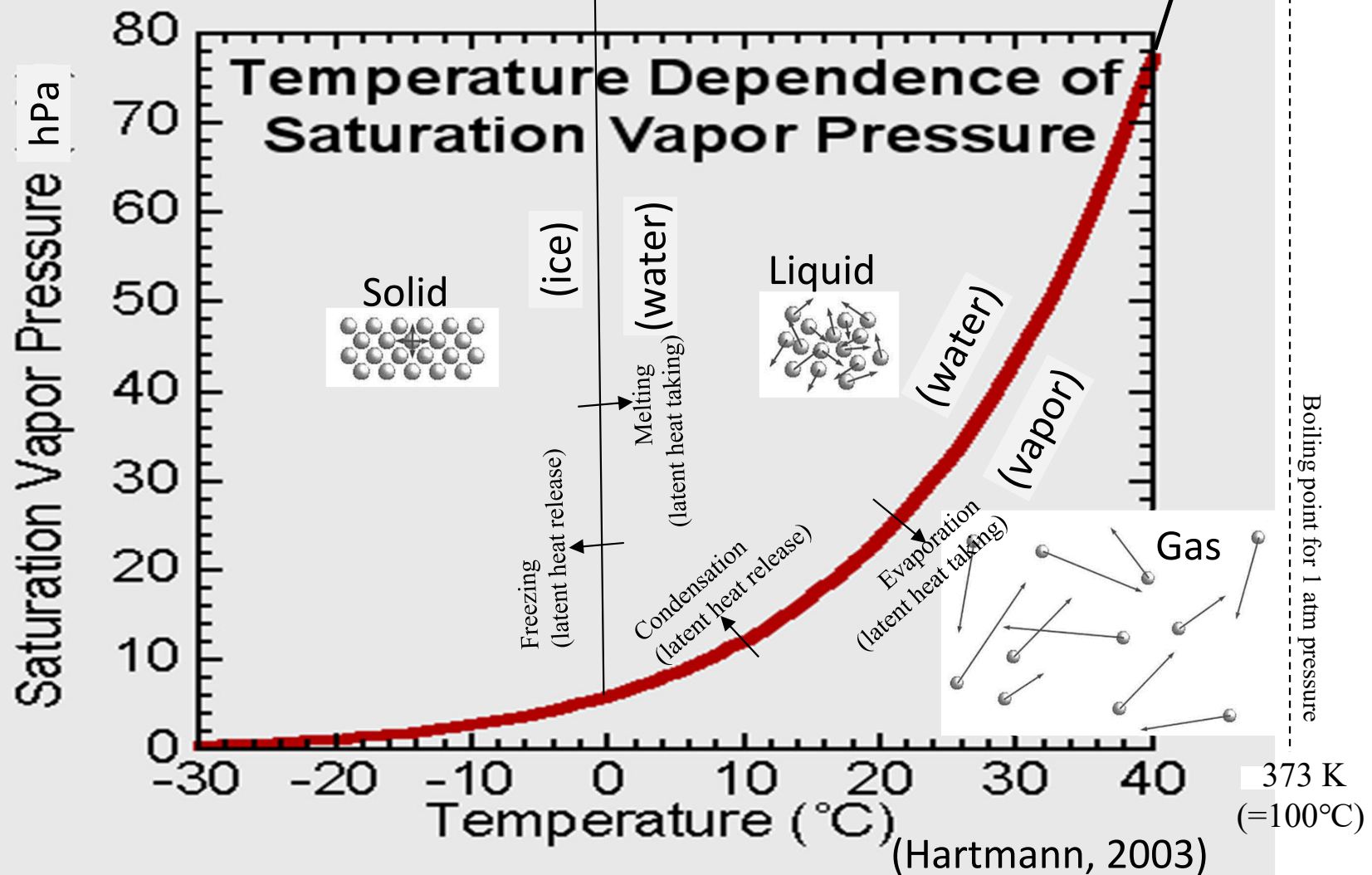
James Prescott Joule
(1818 – 1889)
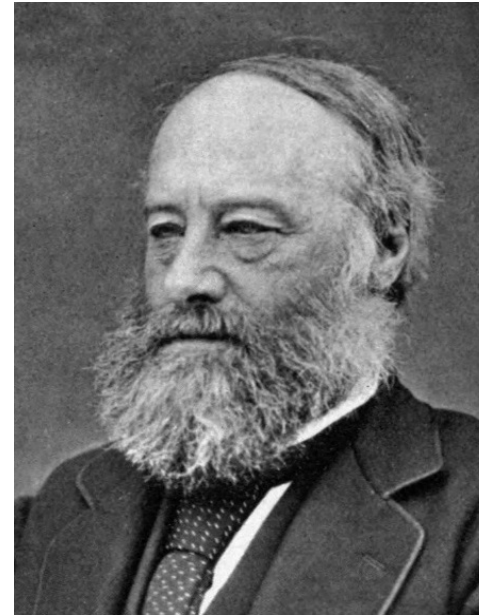
(Roscoe, 1906; https://en.wikipedia.org/wiki/James_Prescott_Joule)

English brewer studied physics without any post at university or institute. He discovered the Joule's law and the mechanical equivalent of heat in early 1840s.

1013 hPa
(= 1 atmospheric pressure (atm))

Relative humidity
= vapor pressure
/saturation pressure

Temperature Dependence of Saturation Vapor Pressure

Saturation Vapor Pressure hPa

80

70

60

(ice) (water)

Solid

Liquid

50

(water)

40

Melting
(latent heat taking)

(vapor)

30

Freezing
(latent heat release)

20

Condensation
(latent heat release)

Evaporation
(latent heat taking)

Gas

10

0

-30   -20   -10   0   10   20   30   40

Temperature (°C)

373 K
(=100°C)

Boiling point for 1 atm pressure

(Hartmann, 2003)

MY1　　MANABU YAMANAKA, 2021/06/14

Temperature units and their definitions

- Fahrenheit (ºF)
  "coolest" (ice + salt) 0ºF
  human body 96ºF
  (now used only in US and Liberia)

Daniel Gabriel Fahrenheit (1686 – 1736)

Polish/Lithuanian
Mercury thermometer

- Celsius (ºC)
  (water at 1 atm pressure 1013.25 hPa)
  freezing point 0ºC
  boiling point 100ºC

Anders Celsius (1701 – 1744)

Swedish astronomer
Earth's ellipsoid

- Kelvin (K)
  (absolute temperature, always positive)
  K = ºC + 273.15
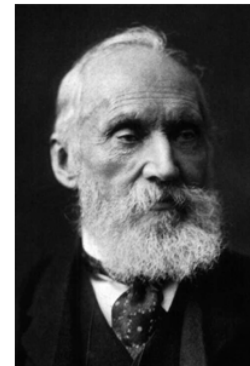
William Thomson, 1st Baron Kelvin (1824 – 1907)

Great British (Scotch) physicist
Thermodynamics, hydrodynamics,
transatlantic cable, ...

Rainbow by liquid droplets

Sunbeams

Unfocused Beam

Focused Rainbow Ray

(Gedzelman)

Figure 5 The primary rainbow is produced by passage of light through the circular cross-section of a spherical raindrop. The rainbow ray is the most focused and least deflected light
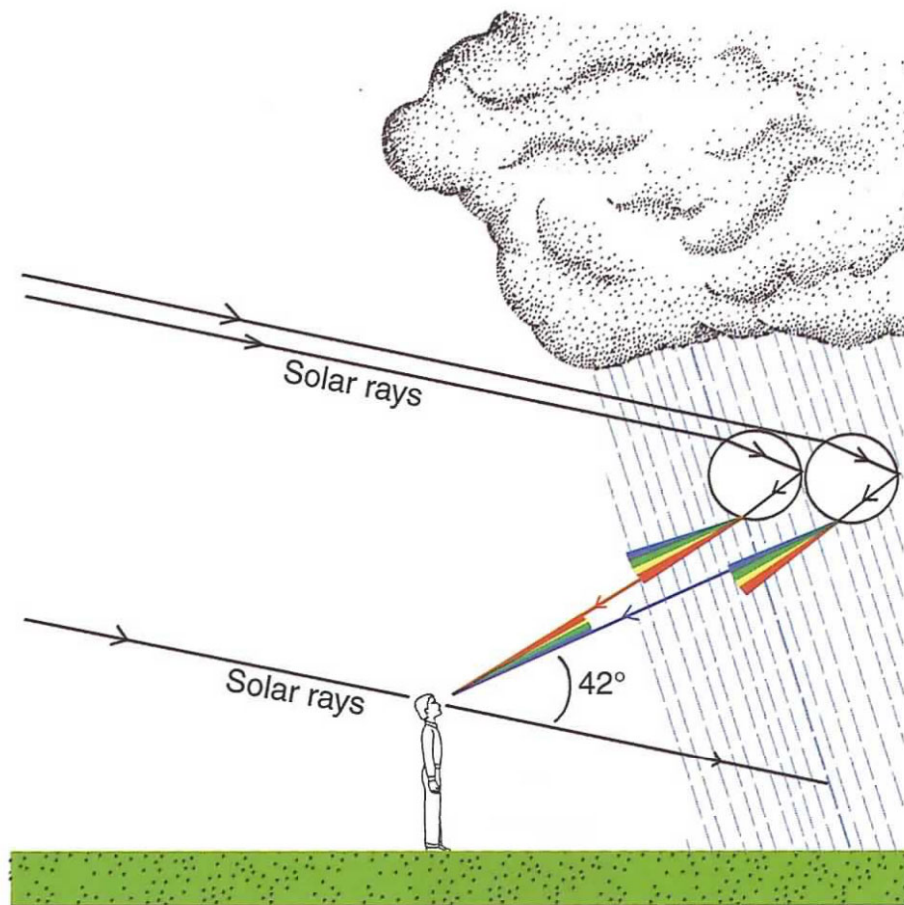
Solar rays

42°

Solar rays

Fig. 4.16 A solar ray is refracted and internally reflected (once) and refracted again by a raindrop to form the (primary) rainbow. The rainbow ray is the brightest and has the smallest angle of deviation of all the rays that encounter raindrop undergo these optical processes. Like a prism, refraction by the raindrop disperses visible light into its component colors forming the rainbow color band. The secondary rainbow is produced by double reflection within raindrops, which appears about 8 degrees above the primary rainbow, with the order of the colors reversed. The rainbow is a mosaic produced by passage of light through the circular cross section of myriad raindrops. (Wallace & Hobbs)
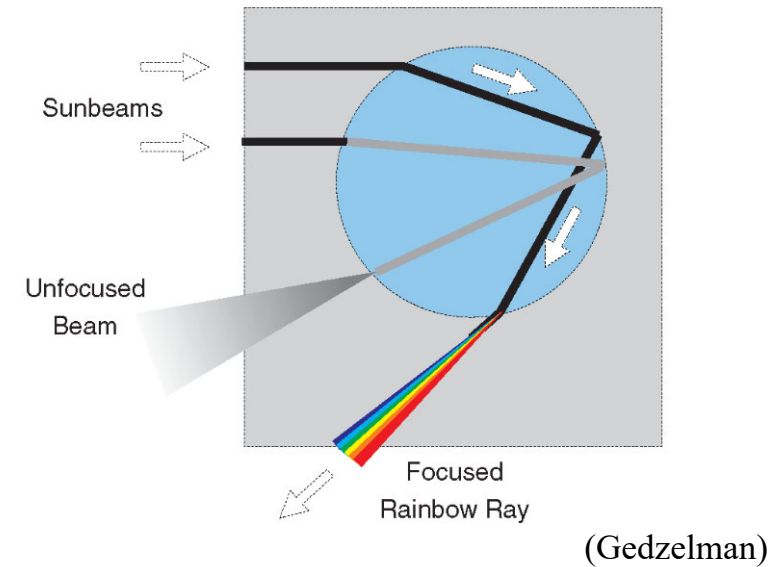
Fig. 4.15 Primary rainbow with a weaker secondary rainbow above it and supernumerary bows below it. [Photograph courtesy of Joanna Gurstelle.] (Wallace & Hobbs)
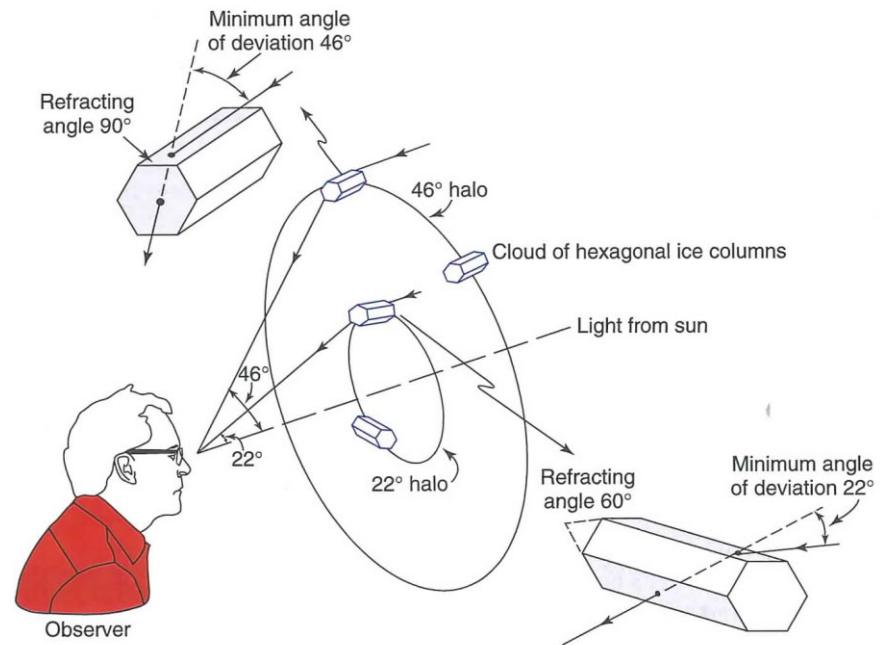
Halo by ice crystals

Minimum angle of deviation 46°

Refracting angle 90°

46° halo

Cloud of hexagonal ice columns

Light from sun

46°

22°

22° halo

Refracting angle 60°

Minimum angle of deviation 22°

Observer

Fig. 4.18 Refraction of light in hexagonal ice crystals to produce the 22° and 46° haloes.

(Wallace & Hobbs)

South Pole halo complex, 2 January 1990. (Photograph courtesy of Walter Tape.)
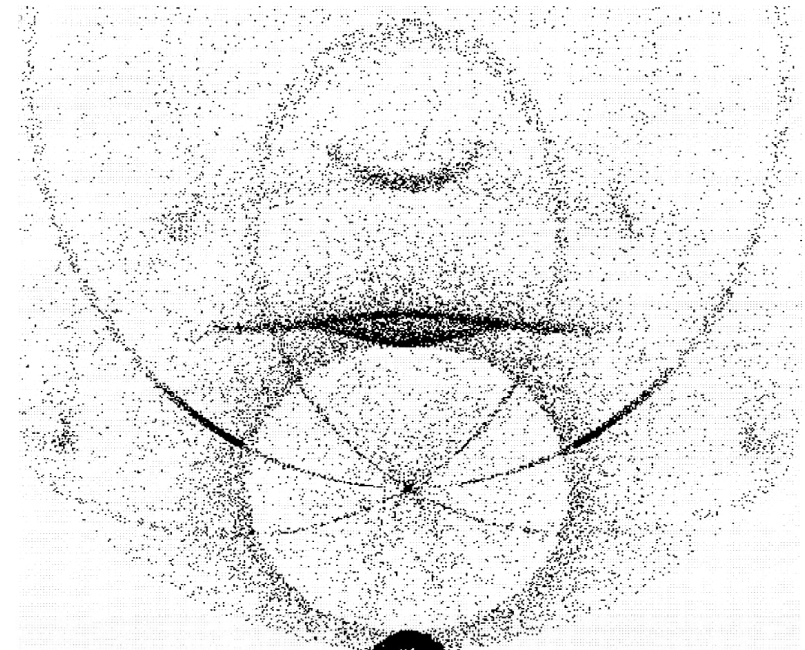
(Gedzelman)

Figure 9 Computer simulation of a complex halo display. Four types of crystals are included. Thick plates falling randomly produce the 22° halo. Thin plates falling almost horizontally produce the parhelia, the parhelic circle, and the circumzenithal arc. Long pencils falling almost horizontally but with random orientation of rectangular sides produce the upper tangent arc to the 22° halo, and the faint supralateral and infralateral tangent arcs to the 46° halo. Long pencils falling almost horizontally with one rectangular face almost horizontal produce the Tape arcs and the Parry arc.

Various fluid flows in the Earth System

[(i) Momentum and (ii) continuity eqs. are common for any cases]

Global atmosphere
(Meteorology)

Ocean
(Oceanography)
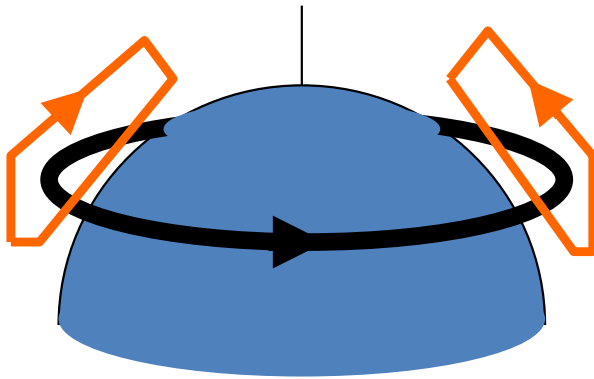
River
(Hydrology)

Compressible
[+thermodynamics (iii)(iv)]
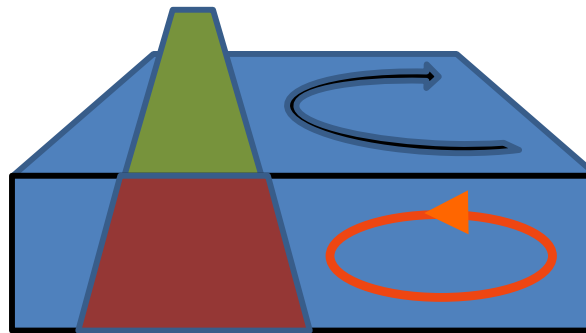
Almost closed

Zonal dominant

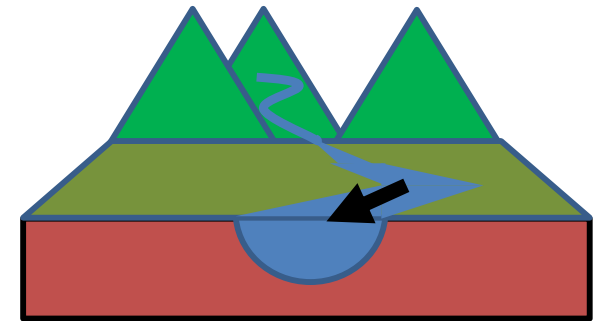Moisture effect [(v)]

Almost free

Incompressible
[+thermal expansion]

Almost closed

Horizontal dominant

Salinity effect

Coastal effect

Incompressible
[+Level/stream change]

Opened

Almost one-dimensional

Complex boundary