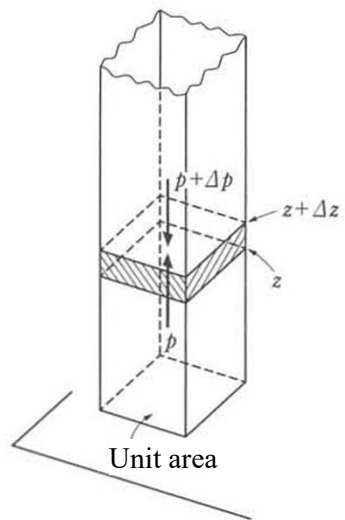


### 3. Atmospheric vertical structure

| Heating             | Radiation                                   | Conduction          | Convection                                      |
|---------------------|---------------------------------------------|---------------------|-------------------------------------------------|
| Heat transfer       | Electromagnetic waves                       | Molecular motions   | Fluid motions                                   |
| Condition           | Including vacuum                            | Continuum           | Gravity                                         |
| Governing law       | Planck's law<br>(Stefan-Boltzmann, Wien)    | Molecular diffusion | Laminar advection<br>Turbulent (eddy) diffusion |
| Temperature profile | dependent on optical depth<br>(exponential) | exponential         | isentropic<br>(upward decreasing)               |
| Constituents        | —                                           | Gravity separation  | Homogenizing                                    |

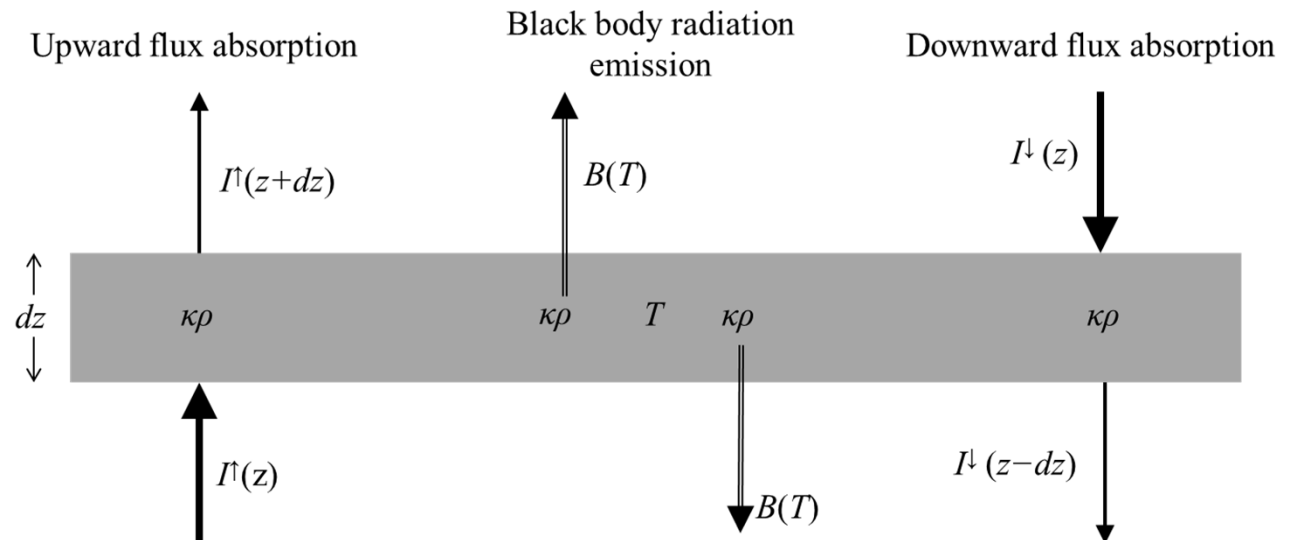
#### Hydrostatic equilibrium



$$\Delta p = -g\rho \Delta z$$

←もつと説明→

#### “Two-Stream” Approximation for “gray” atmosphere



# Vertical distribution of pressure

- Hydrostatic equation:

$$\frac{dp}{dz} = -g\rho$$

- Rewriting  $\rho$  by the ideal gas equation (Boyle-Charles law):  $p = R\rho T$

$$\frac{dp}{dz} = -\frac{g}{RT}p \rightarrow \frac{1}{p} \frac{dp}{dz} = -\frac{g}{RT} \equiv -\frac{1}{H}, \quad H \approx \frac{287 \text{ [J K}^{-1}\text{kg}^{-1}] \cdot 250 \text{ [K]}}{9.8 \text{ [ms}^{-2}]} \approx 7 \text{ [km]}$$

- Differentiation of logarithmic function:

$$\frac{1}{y} \frac{dy}{dx} = \frac{d \log_e y}{dx} = \frac{d \ln y}{dx} \rightarrow \frac{d \ln p}{dz} = -\frac{1}{H}$$

- Integration:

$$\ln p = -\frac{1}{H}z + \text{const.}$$

- Exponential function:

$$p = e^{-\frac{1}{H}z + \text{const.}} = e^{\text{const.}} \cdot e^{-\frac{1}{H}z} \equiv \text{const.'} \cdot e^{-\frac{z}{H}}$$

- Bottom boundary condition:

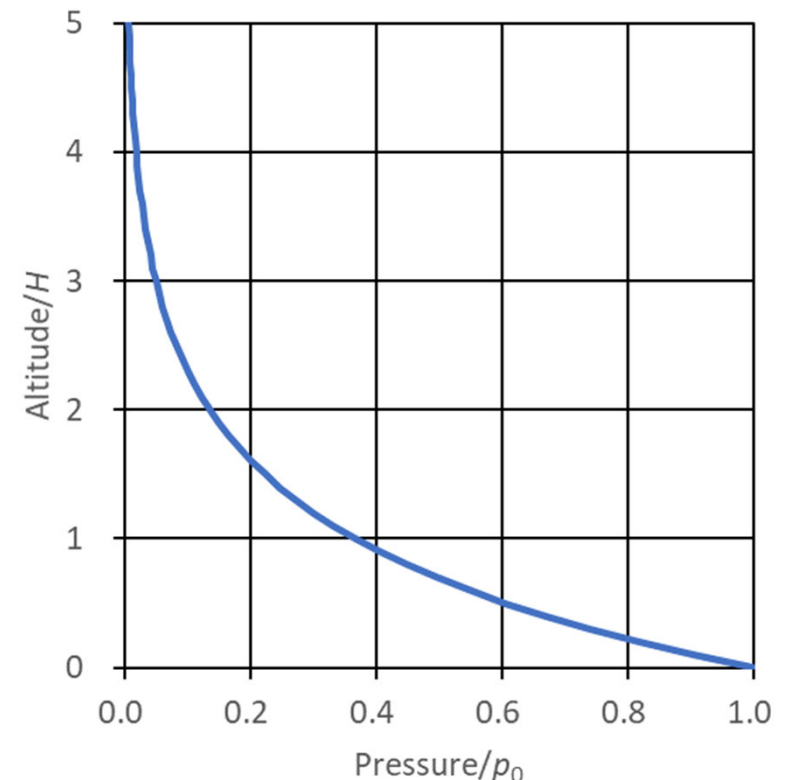
$$p = p_0 \quad \text{at } z = 0 \rightarrow \text{const.'} = p_0$$

- Solution:

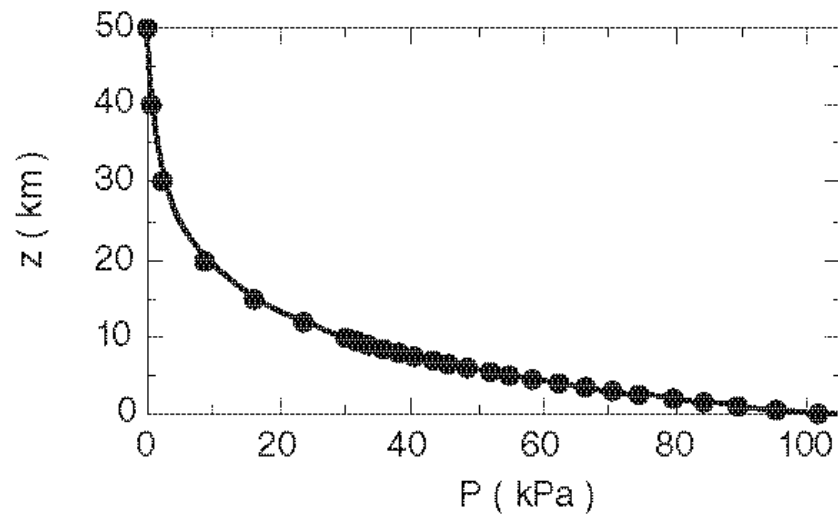
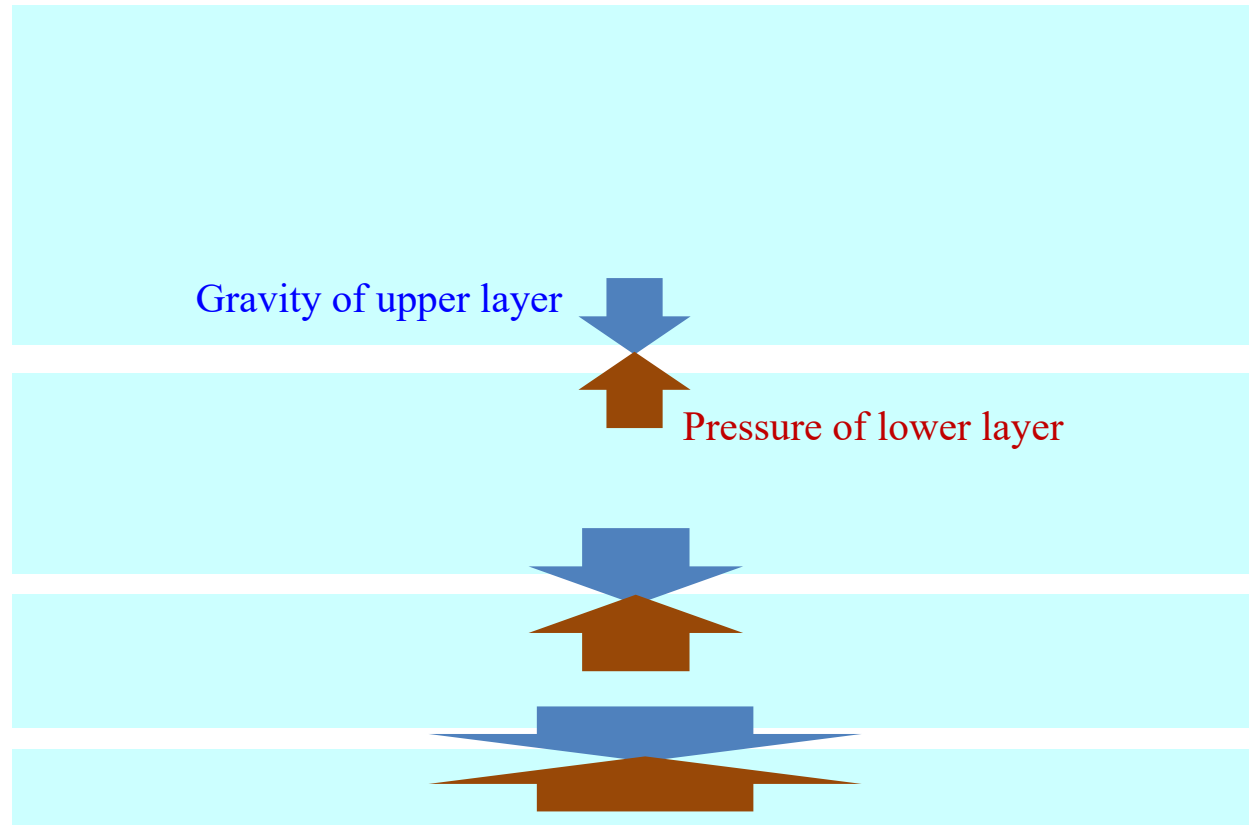
$$p = p_0 \cdot e^{-\frac{z}{H}}$$

- Meaning of the “scale height”  $H$ :

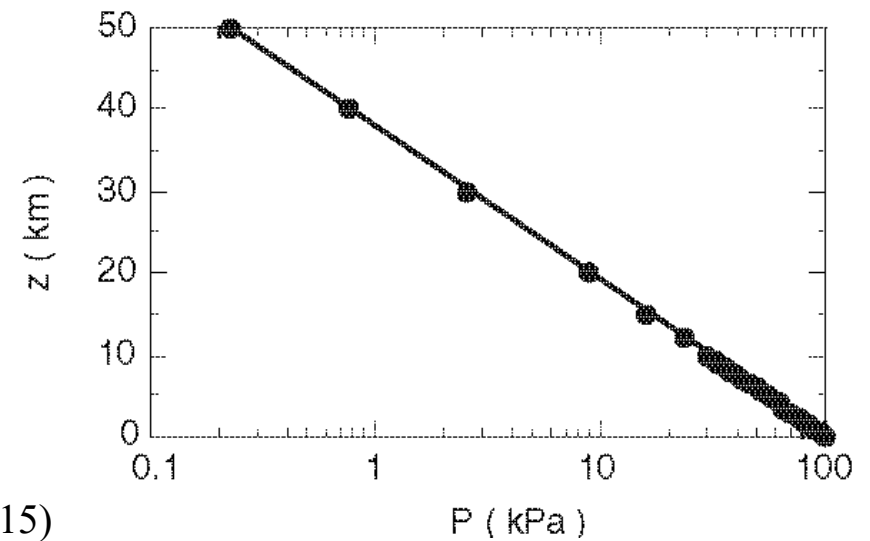
$$p = \frac{p_0}{e}, \frac{p_0}{e^2}, \dots, \frac{p_0}{e^n}, \dots \quad \text{at } z = H, 2H, \dots, nH, \dots$$



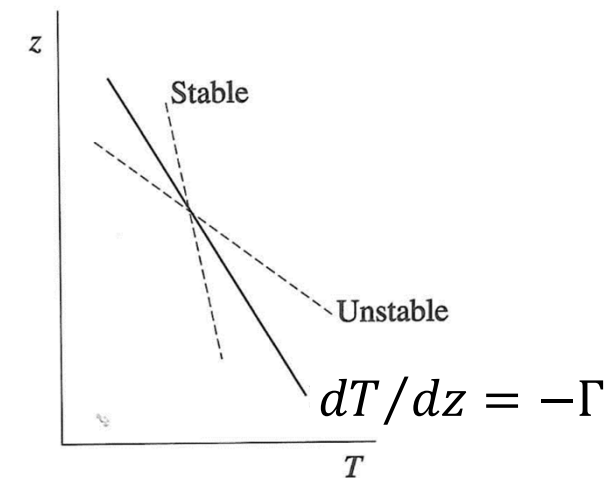
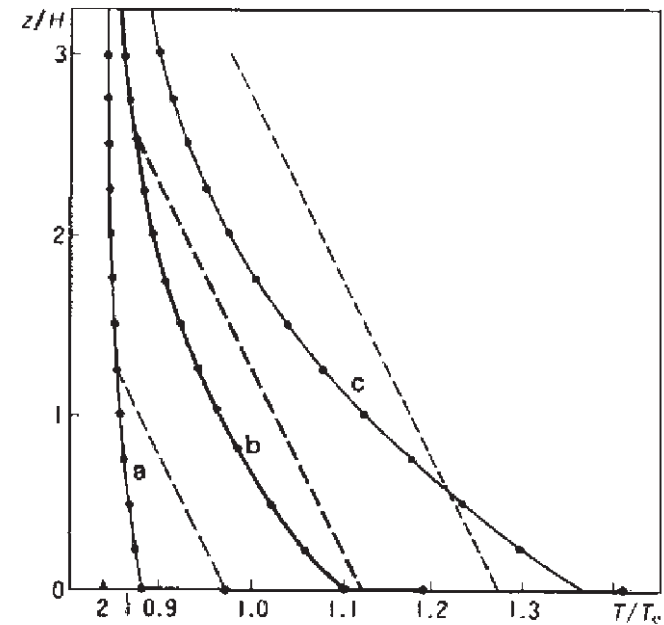
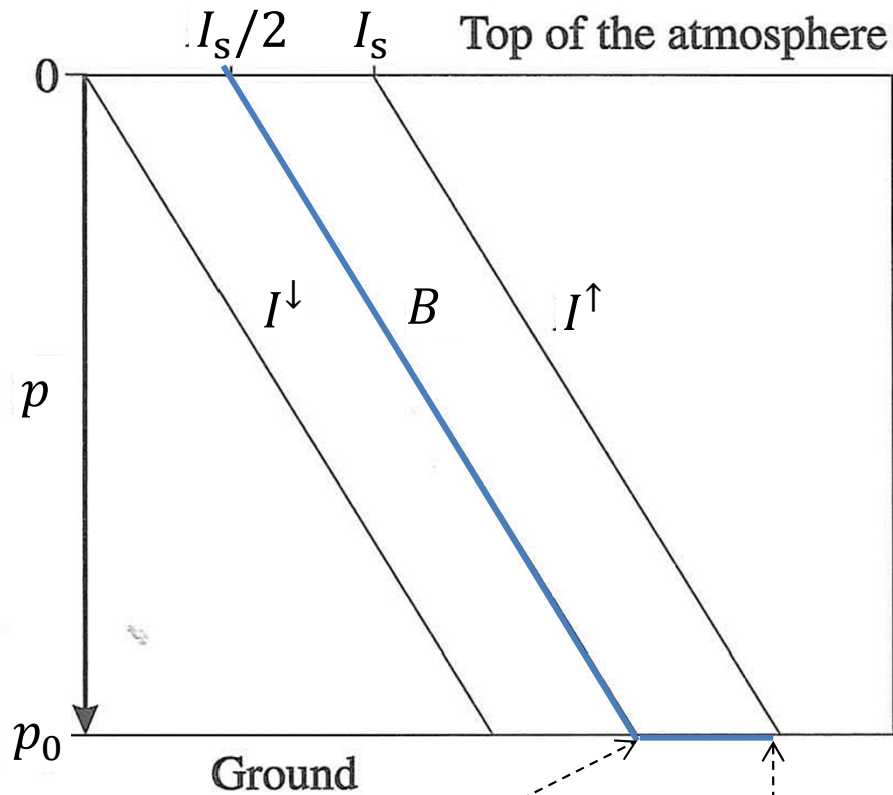
# Pressure upward exponential decrease (Hydrostatic equilibrium)



(Stull, 2015)



$$B(p) = \frac{I_s}{2} \left( \frac{\kappa}{g} p + 1 \right) \longrightarrow T(p) = \left( \frac{B}{\sigma} \right)^{1/4} = \left[ \frac{I_s}{2\sigma} \left( \frac{\kappa}{g} p + 1 \right) \right]^{1/4}$$



Atmos. Temp. at the bottom

$$\left[ \frac{I_s}{2\sigma} \left( \frac{\kappa}{g} p_0 + 1 \right) \right]^{1/4} = \left( \frac{B(p_0)}{\sigma} \right)^{1/4}$$

Ground temperature

$$T = \left( \frac{I^\uparrow(p_0)}{\sigma} \right)^{1/4} = \left( \frac{B(p_0) + I_s/2}{\sigma} \right)^{1/4}$$

# 1-dimensional climate: Radiative-convective equilibrium

- Radiative equilibrium

$$Q = \partial/\partial z [S + I^\uparrow - I^\downarrow] = 0$$

( $S \sim 0$  for  $z \neq 0$ , ozone layer)

$$\Rightarrow \partial/\partial p [\sigma T^4] \sim S/p_1$$

$\Rightarrow -\partial T/\partial z$  increases downward

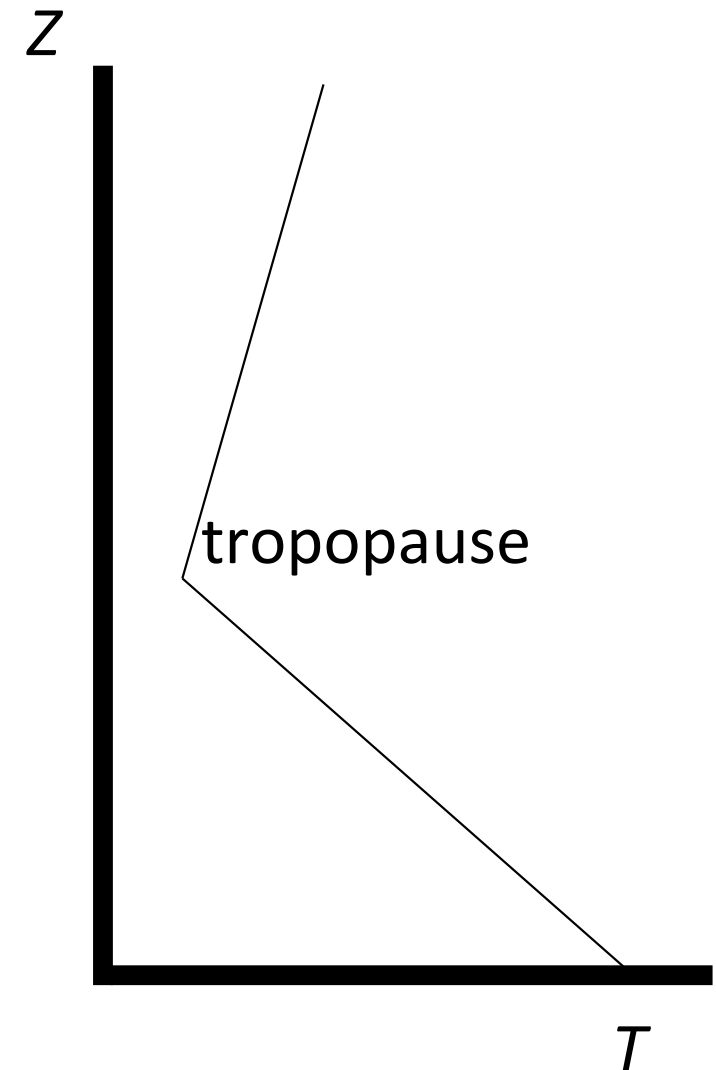
- Convective equilibrium (adjustment)

$$\partial \Theta / \partial z = 0$$

$\Rightarrow$  (dry) adiabatic lapse rate:

$$-\partial T / \partial z = g/C_p \doteq 10 \text{ K/km}$$

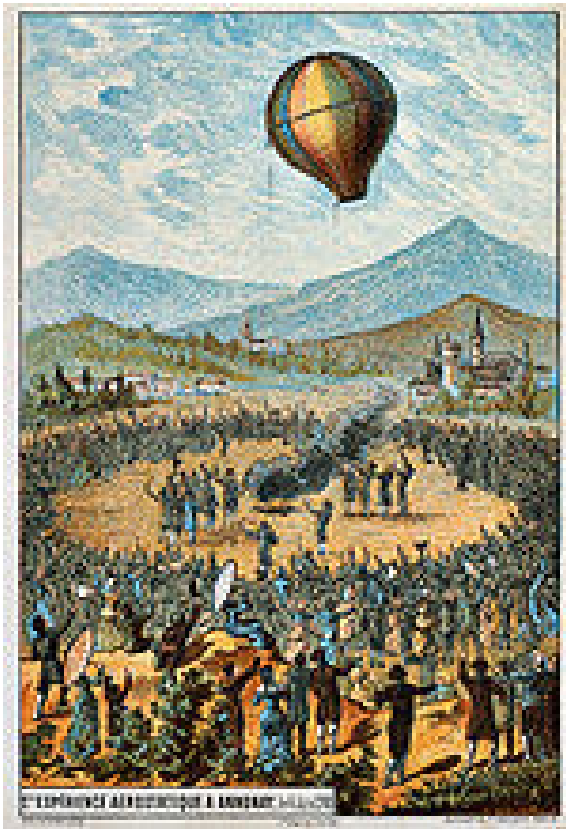
- Separation of stratosphere (radiative)  
and troposphere (convective)



# Balloons, vertical observations



Joseph-Michel Montgolfier (1740 – 1810)  
Jacques-Étienne Montgolfier (1745 – 1799)



Montgolfière (hot air type) balloon  
(at Annonay in June 4, 1783)



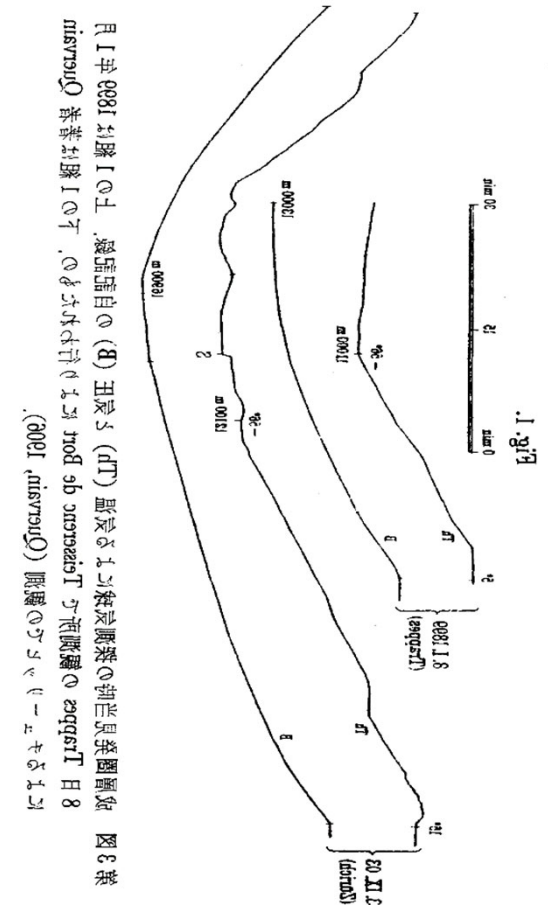
Jacques Alexandre César Charles  
(1746 – 1823)



Charlière (light gas type) balloon  
(at Paris in December 1, 1783)

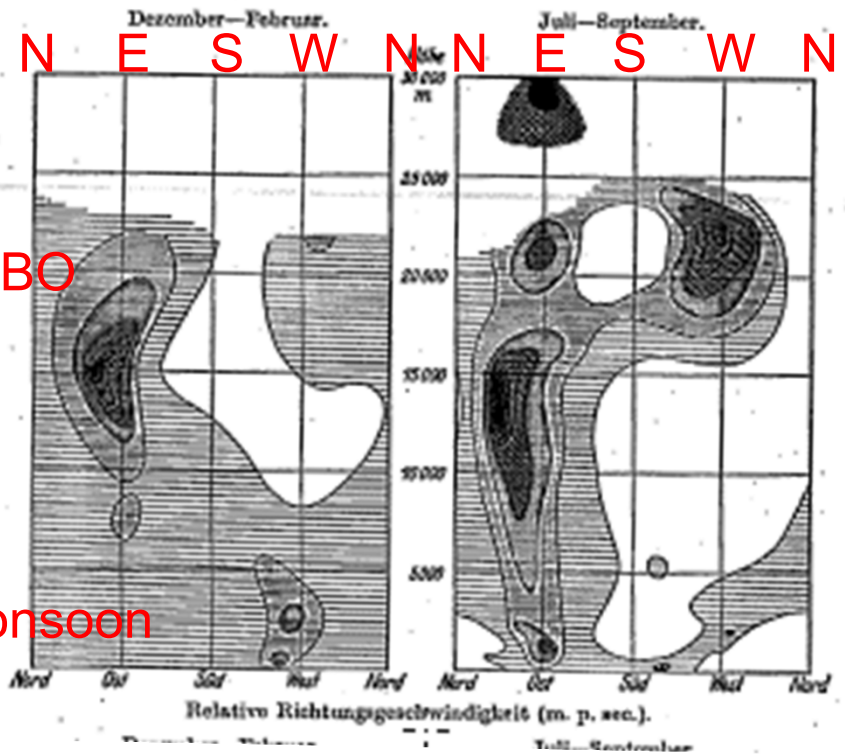
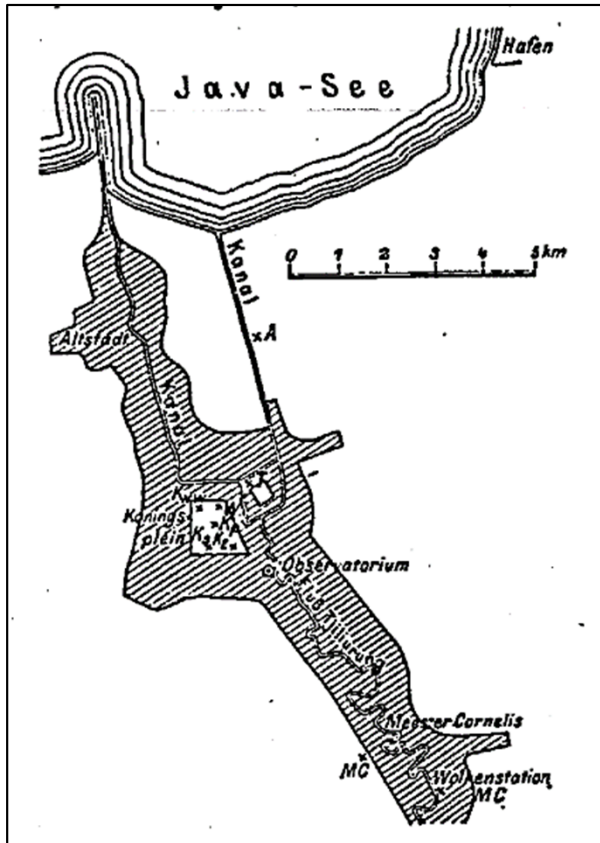
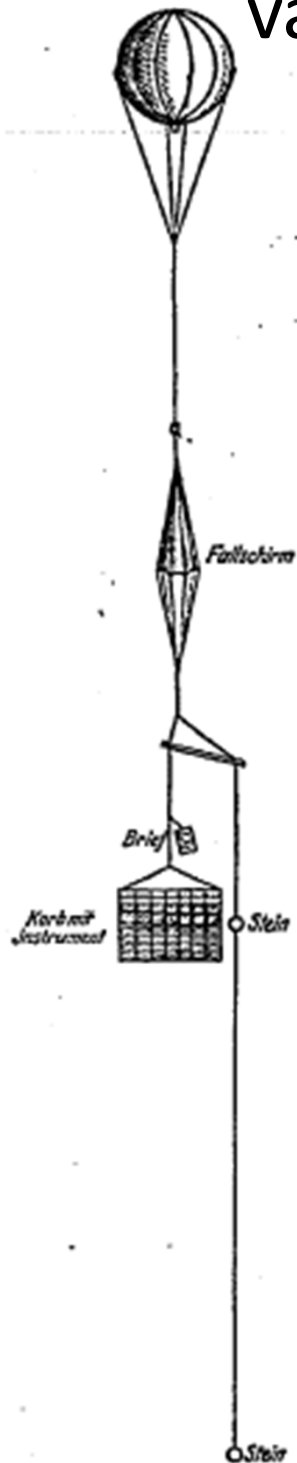


Leon Philippe Teisserenc de Bort  
(1855 – 1913)



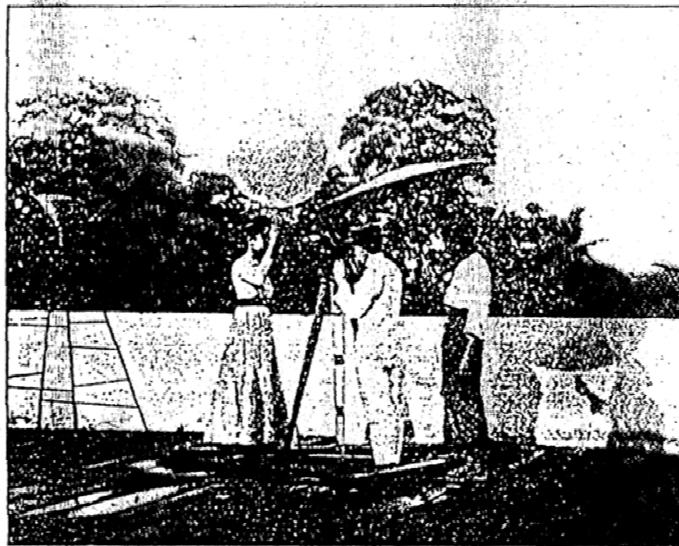
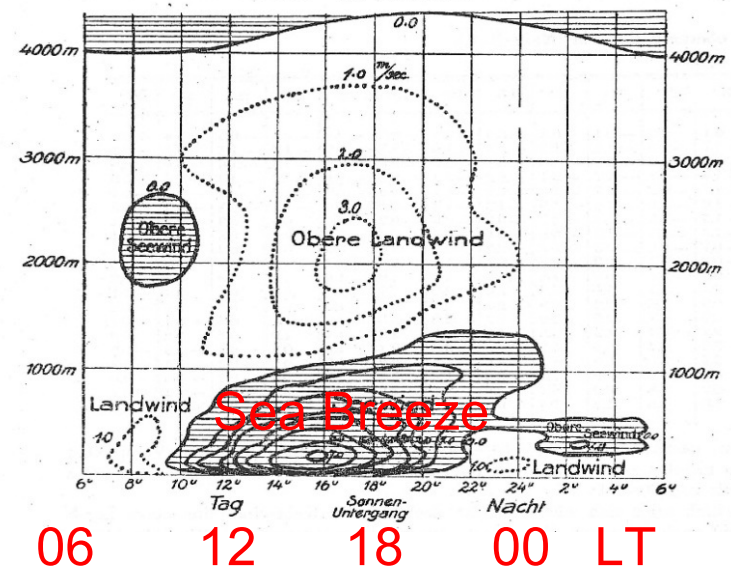
van Bemmelen (1913, 1922)

Rainy (Dec-Feb) Dry (Jul-Sep)



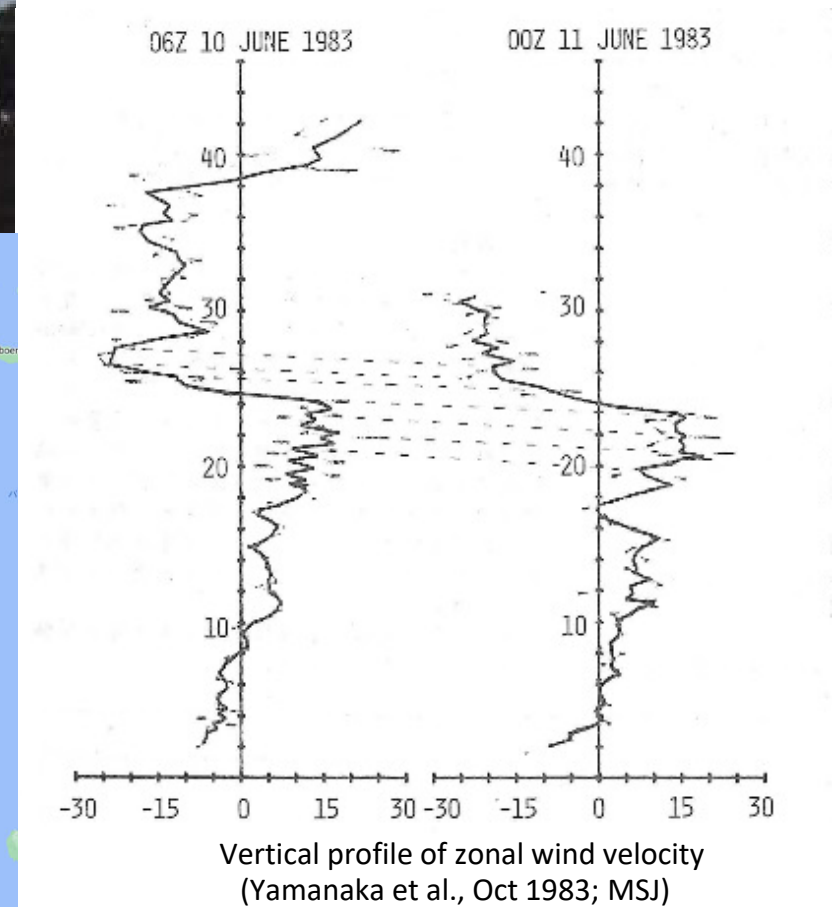
06-24 LT hourly for May-Nov;  
08, 14, 19 LT for Dec-Apr  
during 1905-15

Geschwindigkeits-Isoplethen für  
Land- und Seewind  
in Batavia

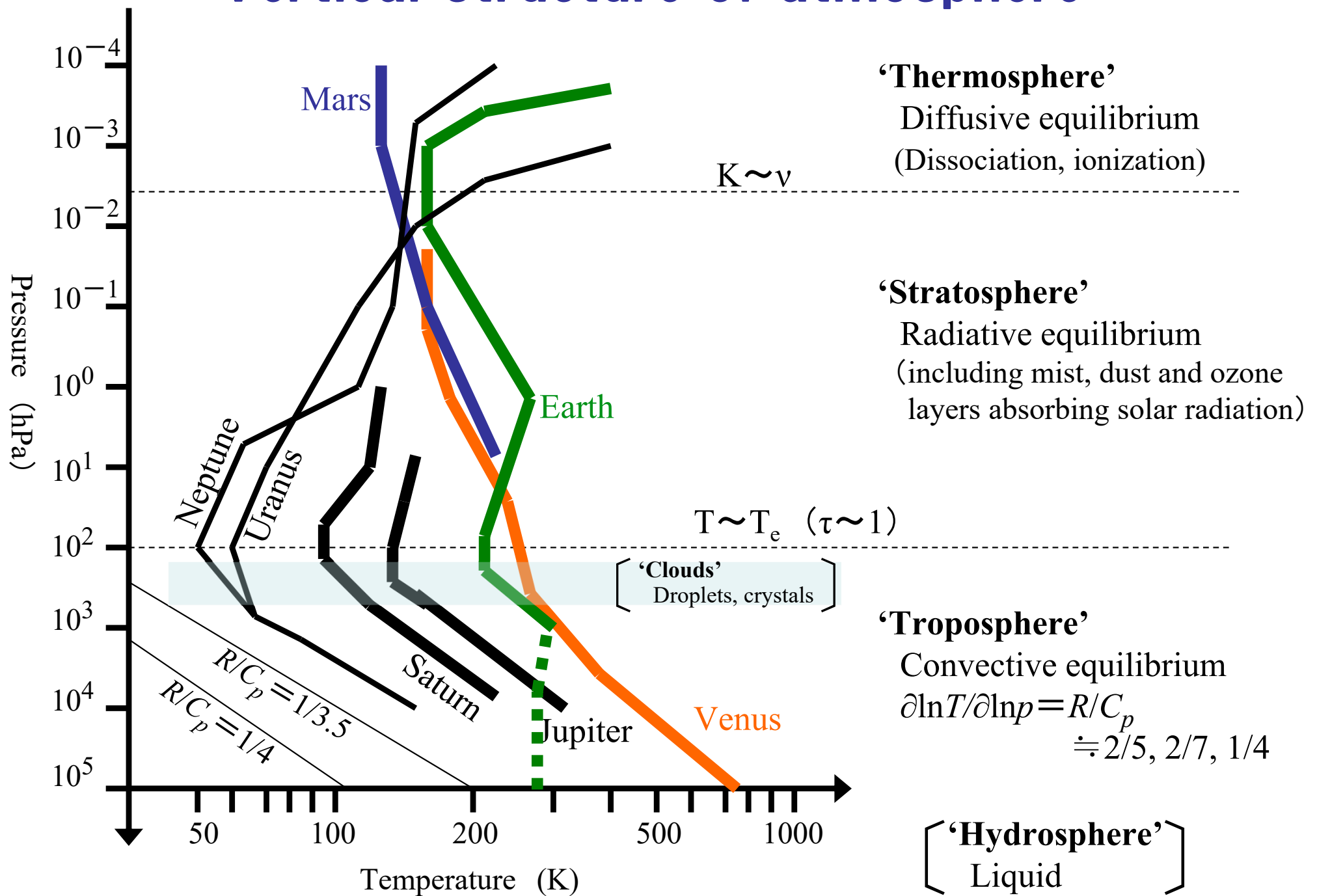


# Balloon observation of Solar eclipse

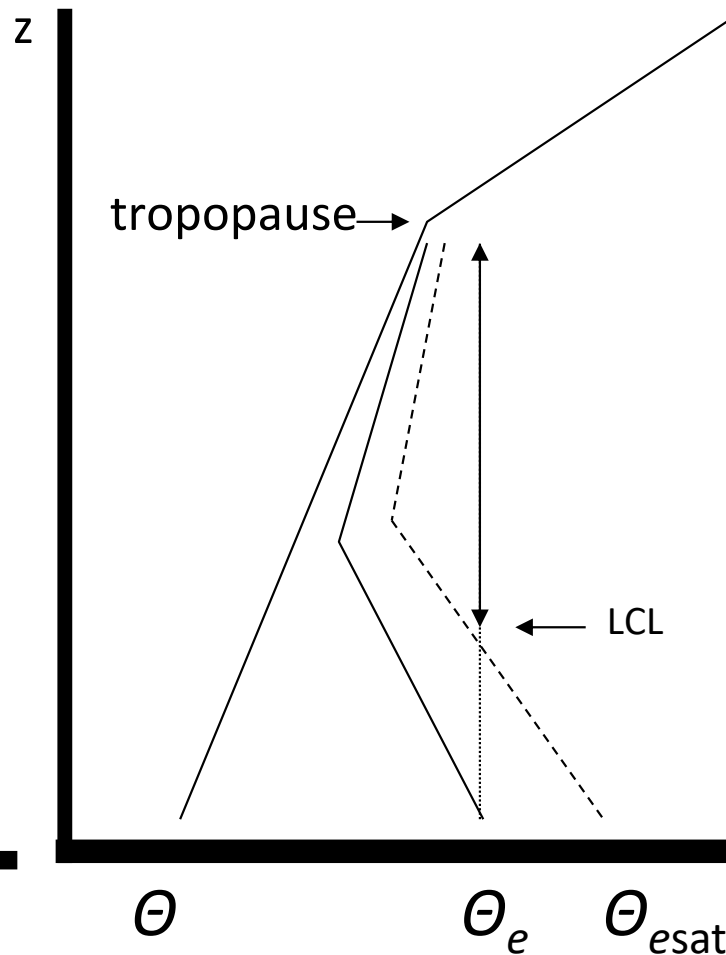
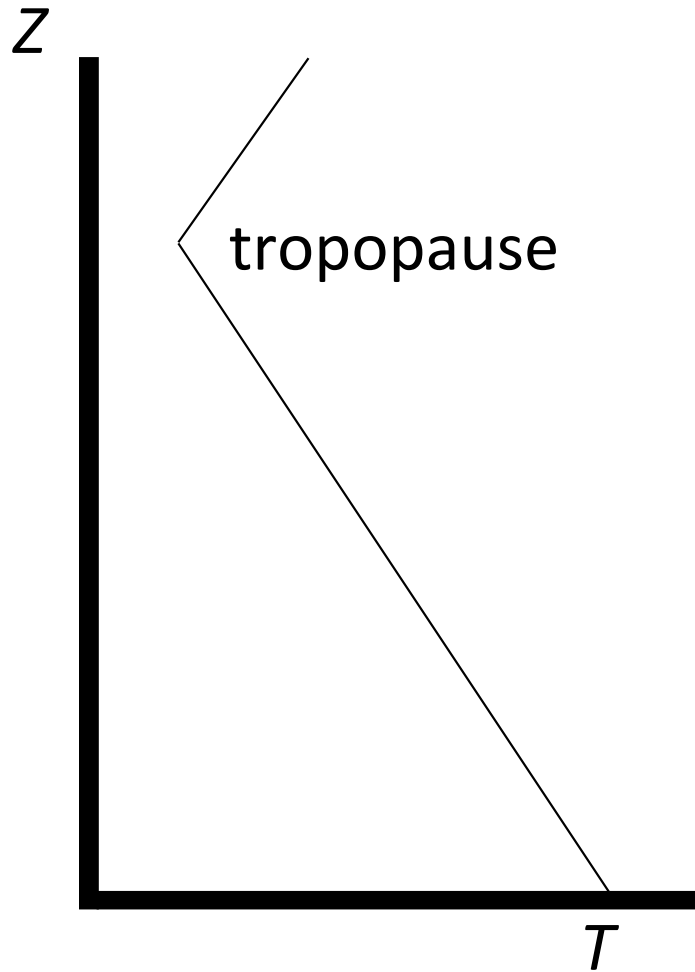
(11 June 1983; Watukosek, JaTim; ISAS, NAO, LAPAN)



# Vertical structure of atmosphere



# Equivalent potential temperature



**(Dry) Adiabatic process:**  
Thermodynamic equation (5)

$$C_p \frac{D\theta}{Dt} = 0 \rightarrow$$

$$\theta = T \cdot \left( \frac{p_{00}}{p} \right)^{R/C_p}$$

**Moist(Pseudo-)adiabatic process:**

(5) and moisture eq. (6):

$$C_p \frac{D\theta_e}{Dt} = LS/T \rightarrow$$

$$\theta_e = \theta \cdot \exp \left( \frac{Lr_s}{C_p T} \right)$$

**“Paradox” of conditional instability:**

- Convection generated spontaneously only when cloud appears.
- Cloud becomes most active when convection is developed.

$\Rightarrow$  *Forced motions (waves, circulations), or CISK*